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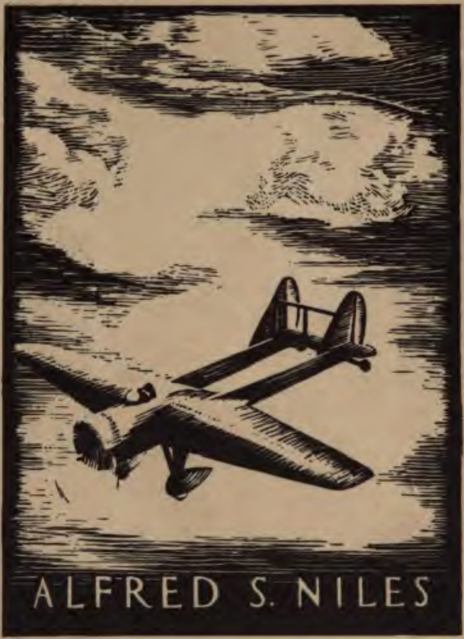
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ESSAY

ON THE

SHAFTS OF MILLS;

CONTAINING

THEIR DESCRIPTION AND USE,

WITH

THE KINDS OF STRESS TO WHICH THEY ARE SUBJECT;

AND

AN INQUIRY

INTO THEIR STIFFNESS, STRENGTH, DURABILITY AND
PROPORTION.

WITH

A VARIETY OF USEFUL TABLES.

ALSO

AN INTRODUCTORY ACCOUNT OF THE PROGRESS
AND IMPROVEMENT OF

MILLWORK.

ILLUSTRATED WITH THREE PLATES.

BY ROBERTSON BUCHANAN,
CIVIL ENGINEER.

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ON THE

SHAFTS OF MILLS.

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ADVERTISEMENT.

Should health and circumstances permit, it is my intention to publish a Series of Essays, all of a practical nature, on Millwork and other Machinery, each forming a whole in itself; at the same time there will be a natural connexion, so that each Essay may again be considered as part of a General System. What I have already published on the Teeth of Wheels, I shall consider as the first of the Series.

PREFACE.

It is my wish to make these Essays useful to operative Mechanics, and to save Engineers and Managers of Manufactories the trouble of much explanation in giving directions to foremen and others, who are to carry their ideas into effect. They will, I hope, give such workmen some notion of the principles on which their work should be conducted; and, however much some of them may be disposed to ridicule theory, yet it has always been found that Machinery erected upon true principles, is ultimately the most economical.

Although I have endeavoured to adapt the style of these Essays to the comprehension of merely operative Mechanics, I trust that men of science will not deem them entirely below their notice, those at least whose situation has precluded them from actual practice.

Persons of this description I have known in great difficulty, from want of information about contrivances which are familiar to every Millwright. Nor is it difficult to account for this, for there is hardly any thing to be found in books with regard to many of the subjects of these Essays. As the path then is almost new and untrodden, I hope to meet with indulgence from the Public.

The following extract from a respectable Periodical Publication, appeared to me so applicable to the subjects of these papers, that I have taken the liberty of transcribing it.*

---

* Eclectic Review, Dec. 1806. Art. XII.

" A country in which Manufactures are extensively established, and conducted with spirit, as in Britain, becomes by degrees a country of Machinery. For inventions to diminish the quantity of human labour employed will be more ingenious in construction, more powerful in operation, and of more general use, in proportion to the necessity of furnishing a greater quantity of commodities at moderate and equable prices. The bodily exertions of workmen in whatever branch of labour have their limits, and excessive efforts, if unduly prolonged, irremediably destroy the health and vigour of those who pursue them. But machines may be continued in activity day and night, week after week, and month after month; having in themselves no life, which suffers a sensible consumption, no principle of activity whose energy requires a pause to effect its recovery or renovation.

"We have seen the Manufactures of our own country solicit the aid of every hand that could be spared from its Agriculture, and seek in distant lands for labourers of every age to supply the mill or to throw the shuttle. We have seen ingenuity exerted to its utmost, to contrive and to construct those machines which these labourers were to superintend and assist. We remember the time when these constructions were the dread and the hatred of the Manufacturers, but we believe the most ignorant workman of the present day acknowledges their utility, and would with difficulty be induced to relinquish that very implement which his father or grandfather would have gladly committed to the flames.

"Considering then the importance of Machines to shorten labour, and the number of persons who are interested in them, as proprietors, as inventors, or as constructors, it is wonderful that so little has hitherto been communicated on this subject by the medium of the Press.

"The process towards perfection in complicated Machinery is perhaps too generally the reverse of what might be expected. When practice has shown the importance of a machine, science takes it up, investigates its principles, analyses its movements, and connects them by the assistance of Mathematical precision.

"Mathematicians are seldom inventors, and workmen are rarely men of science, yet the mutual assistance of study and practice is necessary, to perfect the subject which each is intent on improving." It is my aim then, to come between these two classes, and to make them better acquainted and more useful to each other. How far I have succeeded in the attempt must be left to the determination of time.

In common, with all writers on similar subjects, I have experienced considerable difficulty to find precise technical words to express the different parts of Millwork. Those which are used by Millwrights in different districts being very different from each other. I hope, however, that the explanations, which I have given of the terms, will make them sufficiently clear.

With regard to this particular Essay on the Shafts of Mills, I have treated the subject in a manner similar to that which I followed in the first Essay in the Inquiry into the Strength and Durability of the Teeth of Wheels. For the reasons then given, p. 104, I have not entered into the demonstrations of the elementary propositions which serve to guide in the Inquiry. I have been at pains to collect and arrange facts respecting Gudgeons and Journals in actual use, upon which to ground calculations. This method I consider as being much more certain than grounding calculations on insulated experiments made on the cohesive strength of materials. These, however, I confess to be of great value, and in some cases I have endeavoured to apply them. I hope that the various Tables which I have given in the course of the Essay, will be of great use to the Millwright in finding without trouble the sizes of Gudgeons and Journals for any case that may occur in practice, and that the principles on which they are formed are laid down in so plain a manner that he will easily understand and apply them. These Tables may be considered as certain great lines drawn to guide the Millwright in his work, and even allowing they may not be absolutely true, may find from experience how near they may be approached with safety.

I take this opportunity of acknowledging the advantage which on this, as on former occasions, I have derived from the assistance of my friends.

Glasgow, 10*th* *Dec.* 1808.

AN ESSAY

ON

THE SHAFTS OF MILLS.

INTRODUCTION.

To proportion the diameters of axles to the stress they have to bear, is in Millwork of great practical importance. On the one hand, if the Shafts be made too weak, it is evident they must soon give way; and on the other hand, if made too strong, they occasion not only unnecessary expense in the construction of the machinery, but what is in most cases still worse, a waste of power from unnecessary friction. It is therefore desirable, that the Millwright should have some rules to guide him in this very important part of his business; a part which I believe, has hitherto, in most cases been conducted intirely at random.

I shall endeavour, therefore, in this Essay, to give such a practical view of the subject as shall enable the Millwright to proceed with greater certainty.

Until of late years most of the Shafts used in Millwork were constructed of timber. The use of cast-iron in this and other parts of Millwork, however, has now become almost universal. For this improvement we are perhaps indebted to those who were engaged in the Cotton Manufacture. After Arkwright's invention, it became a great object with them to save time in the erection of Machinery, and to render it as durable as possible; for every stoppage was attended with great loss, by throwing idle the numbers of people necessary in Cotton Mills

Besides the expense attending the repair, what had perhaps still more weight with them was, that the profits at that period on Cotton Spinning, were almost unparalleled in any other branch of manufacture.

Another circumstance which tended very much to the advancement of Millwork, arose from Mr. Watt's improvement of the Steam Engine. This enabled Cotton-spinners and other manufacturers who required power to work their machinery to carry on their business in towns. Hence power and people might, without trouble, be concentrated on the most eligible spot, and the great expense and disadvantages avoided which are attendant on colonizing the remote situations in which powerful falls of water are commonly found. I shall not here enter into the question of health and morals; but certainly the unprecarious supply and steady exertions of such numerous hands, not only rendered the people more expert in all their operations, but tended very much to the improvement of their machinery.

The introduction of cast-iron then may be considered as a kind of new era in the history of Mills. Without the use of this material, it would not have been possible with the same number of operative mechanics to have constructed one tenth of the machinery which has of late years been erected in Great Britain.

CHAP. I.

SECTION I.

General Description of Shafts.

1. The axles used in millwork are commonly denominated, when of a large size, *shafts;* those which are smaller, are usually called *spindles.* Thus, for example, we say the shaft of a water-wheel; the spindle which carries the millstone of a corn-mill.

2. When shafts lie in a horizontal direction, they are called *lying* or *horizontal* shafts; when vertical, they are termed *upright*, or *vertical* shafts.

3. Shafts are usually made of wood, or of iron. Large wooden shafts are generally made either of solid oak, or are built of fir-logs. The scarcity of large oak occasioned the built shafts of fir to come

into more general use. The latest improvement made on wooden shafts, was that of having what are called cross-tailed *gudgeons.** Before that improvement, it was attended with very great trouble and expense, to keep the gudgeons from becoming loose in the shafts. Indeed, it was found impracticable to keep them fast for any considerable time.

4. Fig. 1 represents a wooden shaft, with the gudgeons in use previously to the last improvement. They are called *laid in gudgeons.* ABC is the gudgeon somewhat in the form of the letter T. One of the tails, C, was let into a mortice, and the rest of the gudgeon sunk into its place in the centre of the shaft. In order to accomplish this, it was necessary to cut out the part, from B to D, Fig. 1, No. 2. After the gudgeon was laid in its place, the vacant part was

---

* The *gudgeon* is the arbour or spindle on which the shaft turns.

filled up by the piece of wood DE, Fig. 1, No. 1. The shaft was then hooped, and the end of it driven full of wedges, in order to fasten the gudgeon. This gudgeon is shewn in perspective, Fig. 1, No. 3.

5. Fig. 2 represents a wooden shaft, with cross-tailed gudgeons. This kind of gudgeon is made of cast-iron, and being thin in the cross tails, let in from the end of the shaft, it leaves the wood much more entire than the *laid-in gudgeon*, while its cross-arms take a much firmer hold. After it is let in, the hoops are driven on the end of the shaft, when warm, and lay firm hold of the ends of the cross-tails. The wood is then wedged up, which makes the gudgeons perfectly fast. Fig. 3 is a perspective view of a cross-tailed gudgeon, and Fig. 4 its profile. It is cast with the round part undermost; for which reason, the pattern must have a taper, to make it

rise out of the sand. This taper has, in the cross-tails, another very important use, that of giving the gudgeon the advantage of *dove-tailed joints* with the timber of the shaft when it is wedged up. Instead of wrought-iron hoops, cross-tailed gudgeons sometimes have a hoop of cast-iron cast along with the tails, as represented by Fig. 5.

6. Cast-iron shafts are sometimes made hollow cylinders, and sometimes they are made solid, and of various figures. It is demonstrable, that a hollow cylinder is much stronger, with the same quantity of matter, than it would be if made into a solid of the same length*. This law is very observable in the beautiful economy of nature; for instance, the stalks of plants, the quills of birds, the bones of animals. But, in the works of art, numberless obstacles to perfection continually oc-

---

* See Gregory's Mechanics, vol. 1, page 109.

cur. In this particular case, the expense of making small shafts, hollow, would be very great; and another objection is, the difficulty of making such castings perfect. Shafts of a small diameter are, therefore, commonly made solid.

7. Fig. 6, represents a cast-iron cylindrical shaft. It consists of three parts, the body, ABCD, and the two gudgeons, AEC, and BFDG. The gudgeons are turned, and carefully fitted into the ends of the body, which is bored and turned to receive them. They are then fixed with screw-bolts, which pass through the flanches, as may be seen by inspecting the figure.

This kind of shaft will obviously be variously constructed, according to circumstances. That represented in the figure was made for a cast-iron water-wheel. The use of the small projections HH, &c. is to prevent the eye of the arms from shifting round on the shaft.

When cylindrical shafts are not used, what are called feathered shafts are often adopted.

8. Fig. 7 represents this construction of a shaft. It probably took its name from its resemblance to the *feathered* part of an arrow. It may be here remarked, that shafts of this species, as often constructed, are by no means calculated to withstand the twist brought upon them by the strain of the machinery. From the breadth of the feathers, their strength to withstand lateral pressure, is, no doubt, considerable; but wanting substance between the feathers, they are liable to continual tremour. Where feathers are applied to shafts, I would prefer keeping the body of the shaft fully as strong as the gudgeon, or *journal**, and apply the feathers merely to prevent bending in the middle, as

---

* *Journals*, or *journeys*, are gudgeons subject to torsion.

Fig. 7, No. 4. But the simple square, Fig. 8, is more easily made, and, I believe, has been found in practice, at least as advantageous as any other form that has been tried for solid shafts.

Having given this general account of shafts, I come next to consider the causes from which the stress on them arises.

SECTION II.

Of the Kinds of Stress to which Shafts are subject.

9. There are two kinds of stress to which shafts are liable: first, *lateral stress*, by which they may be broken across: secondly, stress arising from *torsion*, by which they may be wrenched or twisted.

All horizontal shafts are liable to the first kind of stress, viz. lateral stress; and some have no other strain whatever; as for instance, a water-wheel shaft, where the motion is communicated from teeth, on the shrouding. See plate, fig. 9.

In fig. 10, the stress on the upright shaft, arises from torsion only; excepting what may proceed from the inaccuracy of the teeth of the wheels, which, if great, will occasion a considerable lateral thrust.

A vertical shaft, which gives or receives motion, by means of a pulley, has thereby a lateral pressure, brought upon it.

In fig. 11 and 12 the stress is compounded of *lateral pressure*, arising from the weight of the wheels A and B, and that of the shaft itself, and of the torsion or twist produced between the wheels A and B.

The case of lateral stress being the most simple, I shall, in the first place, examine it, confining for the present our attention to the gudgeons only. The bodies of shafts will be afterwards considered.

CHAP. II.

SECTION I.

10. *Of the Strength of Gudgeons, where the Stress is produced by* LATERAL PRESSURE ONLY.

The gudgeons having all the weight on the shaft to support, ought to be made sufficiently strong for that purpose; while, to avoid unnecessary friction, they should be made as small in diameter as possible, consistently with sufficient strength and durability.

When we are able to determine the diameters of the gudgeons, or journals, this serves as a foundation for the proportions of the other parts of the shafts.

Although wrought-iron will bear a greater weight than cast-iron, yet cast-iron being not only cheaper, but much more easily formed into convenient shapes, gudgeons are now most commonly made of that material. We shall, therefore, in the first place, confine our attention to the diameters of gudgeons made of cast-iron. Here it may be proper to state the following proposition:

11. Prop. I.—*Solid cylinders of the same length have their lateral strength as the cube of their diameters**.

That is, if a gudgeon of two inches be sufficient to support a certain weight, a gudgeon of four inches will support eight times as much.

From this law, it is evident, that were all gudgeons made of iron, of the very same quality, knowing the strength sufficient, in any one case it would be easy to calculate what it should be in any other case.

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* See Emerson's 4to. edition, p. 94.

Gregory's Mechanics, vol. 1, p. 107, Cor. 3.

There is, probably, a *typographical error* in Dr. Brewster's edition of Ferguson's Lectures, vol. 2, p. 158, where it is said, "The diameter of the gudgeon must be proportional to the *square root* of the weight which it supports.

But as there is a great variety, in point of strength, in different kinds of iron; Welch cast-iron, for instance, being stronger, as some think, by one-fourth, than that made in Scotland, it is prudent to calculate upon the weakest.* Mr. Banks observes, that, "Iron is much "more uniform in its strength, than "wood; yet it appears that there is "some difference in different kinds of "ore, or iron-stone; there is also a dif-"ference from the same furnace, per-"haps owing to the degree of heat "which it has when poured into the "mould."

I shall proceed in this inquiry, on principles similar to those which I have followed in my *Inquiry respecting the Strength and Durability of the Teeth of Wheels*, namely, by taking a number of cases from millwork in actual use, and drawing inferences from them. This method of making inferences, by collecting and arranging facts respecting millwork, I consider as being much more safe and useful, than founding calculations upon experiments, often made on a small scale, and under circumstances very different from those which occur in practice with machinery. I shall begin with considering the gudgeons of water-wheels.

---

* Banks's Power of Machines, p. 94.

SECTION II.

Of Gudgeons of Water-wheels.

12. In the following table, water-wheels of various weights are collected, and the diameters of the gudgeons in actual use, stated. The weights of the cast-iron wheels were found, from the

weight of the castings, &c. of which they are composed. The wooden wheels are estimated, making allowance for the wood becoming heavier by being soaked in water. I am aware, however, that besides the mere weight of the wheel, other circumstances should be taken into account,* such as the weight of water in the buckets, and the pressure brought on the gudgeon by the resistance of the work, &c. But I shall follow the general result of those cases only, in which the gudgeons have been found sufficiently strong, and, I apprehend, that those extraneous causes on the one hand, will not affect our rules more than the different qualities of cast-iron will the strength of gudgeons on the other hand.

The water-wheels in the table were all in the middle of their respective shafts. Both gudgeons of each wheel had, therefore, equal stress.

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* See Encyclopedia Britannica, article Rotation.

13. *Description of the first Table of Gudgeons of Water-wheels.*

Column 1 Contains letters to distinguish the wheels.
——— 2 Shows the material of which the wheel is made.
——— 3 The diameter in feet.
——— 4 The width in feet.
——— 5 The kind of wheel.
——— 6 The diameter of the gudgeon in inches.
——— 7 The weight of some of the wheels, in tons and cwts.
——— 8 The weight of some of the wheels, in cwts. and qrs.
——— 9 Contains the cube root of the weight.

The use of this column is to compare the several gudgeons with the law contained in Prop. 1. For, were all the gudgeons duly proportioned to the weight they have to sustain, they would be to one another as the cube roots of their weights.

TABLE I. GUDGEONS OF WATER-WHEELS.

1	2	3	4	5	6	7		8		9	
	Wheel, made of	Diameter of wheel, in feet.	Width in feet.	Kind.	Diameter of cast-iron gudgeon in inches.	Weight of wheels, in tons and cwts.		Weight of wheels, in cwts. and qrs.		Cube Roots of weight, in cwts.	Remarks.
A	Wood	24	12	Overshot	7	23	14	474		7.796974	
B	Cast-iron	16	6	do.	7	12		240	3	6.214464	
*C	Cast-iron	16	8	do.	6⅜	16	10	330		6.910423	This Gudgeon broke from being a bad casting.
D	Cast-iron wheel and buckets	32	4½	do.		24		480		7.829735	
E	Wood	16	9	do.	6	10	11	211		5.953341	
F	Wood	12½	7	do.	6	5	14	114		4.848808	
G	Wood	32	11	do.	10						
H		21	10	do.	7						Broke, owing to their too great length, they were 11 inches long.
I	Wood	28	6	do.	8						
K	Fir	18	10	do.	10						

* The hollow shaft of C, being 16 inches diameter, and 2 inches thick, broke, in consequence of the gudgeon getting loose about 7 March, 1808.

Observations on the first Table of Gudgeons.

14. Particular care should be taken, that the axis of the gudgeons be exactly in a line with the axis of the shaft which it supports, otherwise the motion will be unequal, and at one part of the revolution the stress will be thrown to the point of the gudgeon; this would endanger its breaking, more particularly if very long, though otherwise sufficiently strong. In the case H, we have an instance of a gudgeon breaking, from being made too long. In practice it is a good method to turn the gudgeons of a wooden shaft, after they are fixed in their places, a second time; in order to render them quite true.

15. From comparing the diameters of the gudgeons (column 6) with the cube root of the weight in cwts. of the wheel (col. 9), it will be found that they approach one another. In other words, the cube root of the weight in cwts. is nearly equal to the diameter in inches. In the

case C, the gudgeon broke, only from being a bad casting, although it is smaller in proportion to its weight, than most of the other cases. Since it was renewed of the same size, it has continued to support its work. When, therefore, we can ascertain the weight of a water-wheel, we have a very simple rule for finding the diameter which the gudgeon ought to have.

RULE I.

The CUBE ROOT *of the weight of a water-wheel, in* HUNDRED WEIGHTS, *is* NEARLY *equal to the* DIAMETER *in inches of a cast-iron gudgeon* SUFFICIENTLY STRONG *to support such wheel.*

I say *nearly*, it being evidently most prudent to make the gudgeon a little more than less in diameter, and to make due allowance for wearing.

EXAMPLE.

Suppose a water wheel to weigh 12 tons, 0 cwt. 3 qrs. what ought to be the diameter of a cast-iron gudgeon, sufficiently strong to support the wheel?

12 tons =240 *cwt.* 3 *qrs.*

The cube root of 240.3 = 6.214464 *Answ.* That is, the diameter of the gudgeon should not be less than $6.\frac{2}{10}$ diameter. It ought to be rather more, to allow for wearing, &c. See B. in the first Table of Gudgeons.

(16.) As the weights of wooden water wheels cannot be accurately known, without a good deal of calculation, it is desirable to have some more ready method for practical purposes.

The weights of overshot, or bucket waterwheels, will be to one another nearly as their circumferences, or diameters and breadth.—I say *nearly*, because the arms will make large wheels heavy in rather a greater proportion. Hence the following Rule is formed on the direct proportion of the sole and buckets adding one-half of the diameter increased in the duplicate ratio or square of the diameter.

RULE II.

For wooden water wheels, multiply the diameter in feet by the width also in feet, to which add the square of half of the diameter. The cube root of the sum will be nearly equal to the diameter of the gudgeon in inches.

Explanation of Table II of Water Wheels.

All the columns, except No 10, are the same as in Table I. The column 10 shows the result by Rule II.

EXAMPLE.

Suppose a wooden water wheel 12 feet diameter and 7 feet wide (See E in Table II of Gudgeons).

12 × 7 = 84
The square of 6 = 36

120 = The cube root of which = 4.932424; that is the gudgeon should not be less than about 5 inches diameter.

TABLE II. GUDGEONS OF WATER-WHEELS.

	Wheel made of	Diameter of wheel in feet.	Width in feet.	Kind.	Diam. of cast-iron gudgeon in inches.	Weight of wheel in tons.		Wt. of wheel in cwts.	Diamr. of gudgeon by calculating the weight.	Diam. of gudgeon of wooden water whs. by Rule II.	Remarks.
A	Wood	24	12	Overshot	7	23	14	474	7.796974	7.559525	
B	Cast-iron	16	16	do.	7	12	2	242	6.231678		
C	Cast-iron	16	8	do.	6⅞	16	10	330	6.910423		
D	do. wheel and buckets	32	4½	do.	9	20		400	7.368063		
E	Wood	12	7	do.	6	5	14	114	4.848808	4.932424	
F	Wood	32	11	do.	10					8.471647	
G	Wood	21	10	do.	7					6.839903	
H	Wood	28	6	do.	8					7.140037	
I	Wood	16	9	do.	6	10	11	211	5.953341	5.924991	
K	Wood	18	10	do.	10					6.390676	

SECTION III.

Of Cast-iron Gudgeons for various Purposes.

(17) Taking it for granted that the cube root of the weight of a water wheel, in hundred weights, is nearly equal to the diameter in inches of a cast-iron gudgeon, sufficiently strong to support such wheel, the following Table of the diameters of gudgeons, and the weights which they may be supposed to sustain, is formed. It may be of use for finding the diameters of journals in all cases of stress arising from lateral pressure, such as grindstones, intermediate spindles, &c. where the pressure can be ascertained, as well as water wheels.

Explanation of the Table of Cast-iron Gudgeons.

Column 1 Contains the diameter in inches, from 1 to 11 inches.

Column 2 Contains the cube of that diameter, or the hundred weights which the gudgeon may sustain.

N. B. I have already remarked (No. 15), that it would be most prudent to make the gudgeon a little more in diameter than the cube root of the hundred weights.

TABLE OF CAST-IRON GUDGEONS.

Diameter in inches.	Cube of Diameter or cwts. which the gudgeon may sustain.
1.	1.
1.25	1.953125
1.5	3.375
1.75	5.359375
2.	8.
2.25	11.400625
2.5	15.625
2.75	20·796875
3.	27·
3,25	34·328125
3,5	42·875
3.75	52·734375
4.	64.
4,25	76·765625
4,5	91·125
4.75	107·171875
5.	125·
5.25	144.703125
5.5	166.375
5.75	190.109375
6.	216.
6.25	244.140625
6.5	274.625
6.75	307.546875
7·	343.
7·25	381.078125
7·5	421.875
7·75	465.483375
8·	512.
8·25	561.515625
8·5	614.125
8·75	669.921875
9·	729.
9·25	791.453125
9·5	875.375
9.75	926.859375
10.	1000.
10.25	1076.890625
10.5	1157.625
10.75	1242.296875
11.	1452.

USE OF THE TABLE.

EXAMPLE I.

Suppose a cast-iron gudgeon of 5¾ diameter, what weight of a water wheel would it be capable of sustaining?

In the first column find 5.75. Opposite to which will be found 190.109375 hundred weights, which is rather more than 9½ tons, which is the answer. But in practice, it would perhaps be prudent not to load this gudgeon with more than nine tons.

EXAMPLE II.

Suppose a grindstone weighing 15½ hundred weights, required the size of a gudgeon sufficient to sustain this weight. Look in the second column for the nearest weight to that given, which will be found to be 15.625, opposite to which, in the first column, is 2½ inches, the diameter of the gudgeon required.

But in practice the spindles of grindstones are commonly of wrought iron, which metal I shall presently consider as applied to gudgeons.

SECTION IV.

Of Malleable or Wrought-iron Gudgeons.

(18) Professor Robison states*, that the cohesive force of a square inch of cast iron is from 40.000 to 60.000lb. wrought iron from 60.000 to 90.000lb.

In the year 1795, I had occasion to substitute cast-iron gudgeons for those of wrought iron, and made some experiments on those metals, from which I drew the following inference: *that gudgeons of the same size, of cast and of wrought iron in practice, are capable, at a medium, of sustaining weights without flexure, in the proportion of* 9 *to* 14.

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* Encyclopedia Britannica, article Strength of Materials, 40.

Taking it for granted that this proportion is near the truth, we may find the diameter which any wrought-iron gudgeon ought to have when its lateral pressure is given, in the following manner:

1. Find the diameter which a cast-iron gudgeon should have to sustain the given pressure, then say, as 14 is to the cube of the diameter of the cast-iron gudgeon, so is 9 to the cube of the diameter of the wrought-iron gudgeon.

The root of this last number gives the diameter required of the wrought-iron gudgeon.

EXAMPLE.

Suppose the lateral pressure to be 125 hundred weights, the cube root of which is 5, the diameter in inches of the cast-iron gudgeon: then say,

As . . 14
Is to . 125,
So is . 9
To 80.357,

The cube root of which is 4.30887.

Upon this principle the following Table is calculated to show the proportionate diameters of cast-iron and wrought iron gudgeons.

EXPLANATION OF THE TABLE

Of Cast-iron and Wrought-iron Gudgeons.

Column 1 and 2	are the same as those in the table of cast-iron gudgeons.
Column 3	Contains numbers in the proportion of 9 to 14 less than those in column 2.
Column 4	Contains the cube root of column 3, or the diameters of wrought-iron gudgeons, having the same strength as those of cast-iron in column 1.

USE OF THE TABLE.

EXAMPLE.

To find the diameter of a wrought-iron gudgeon of the same strength with one of cast-iron of 3 inches diameter. Look on the 1st column for 3, and on the same line in the 4th column will be found 2.571282, that is, a little more than 2½ inches, the diameter required of the wrought-iron gudgeon.

The numbers in the 3d column, being the cube of those in the 4th, another use may be made of this part of the table. For, supposing the 4th column to represent cast-iron gudgeons, then the 3d column will represent the hundred weights which cast-iron gudgeons of those diameters should sustain.

TABLE OF CAST AND WROUGHT-IRON GUDGEONS.

1	2	3	4
Diamr. of gudgeons in inches.	Cube of Diamr. of cast-iron gudgeon, or the cwts. which the gudgeons may sustain.	Cube of diamr. of wrought iron gudgeons.	Diamr. of wrought iron gudgeons in inches and parts.
1.	1.	.6428571	.6428571
1.25	1.953125	1.2555803	1.063340
1.5	3.375	2.1696427	1.259921
1.75	5.359375	3.4453125	1.514825
2.	8	5.1428571	1.709976
2.25	11.400625	7.3289732	*1.912933
2.5	15.625	10.0446428	2.164435
2.75	20.796875	13.3694196	2.351335
3	27.	17.3571428	2.571282
3.25	34.328125	22.0670803	2.802039
3.5	42.875	27.5625	†3.018294
3.75	52.734375	33.9006696	3.239612
4	64.	41.1428571	3.448217
4.25	76.765625	49.3493303	3.659306
4.5	91.125	58.5803571	3.881936
4.75	107.171875	68.896	4.101566
5	125.	80.357	4.308870
5.25	144.763125	93.023	4.530655
5.5	166.375	106.955	4.747459
5.75	190.109375	122.213	4.959675
6	216.	138.857	5.180101
6.25	244.140625	156.948	5.394690
6.5	274.625	176.545	5.609376
6.75	307.546875	197.709	5.828476

* Dr. Brewster says, " a Gudgeon 2 inches diameter, will sustain 3239lb. averdupois." Brewster's Edition of Ferguson's Lectures, vol. i. p. 157.

† The wrought iron Grindstone spindle used by Mr. Southern in his experiments on Friction, was 3$\frac{1}{8}$ diameter, and carried a stone weighing 3700lb. See Phil. Mag. Vol. xvii. 123.

Table of Cast and Wrought-Iron Gudgeons, continued.

1	2	3	4
Diamr. of gudgeons in inches.	Cube of Diamr. of cast-iron gudgeon, or the cwts. which the gudgeons may sustain.	Cube of diamr. of wrought iron gudgeons.	Diamr. of wrought iron gudgeons in inches and parts.
7.	343.	220.500	6.041377
7.25	381.078125	244.979	6.257324
7.5	421.875	271.205	6.471274
7.75	465.484375	299.240	6.686882
8.	512.	329.143	6.903436
8.25	561.515625	360.975	7.120367
8.5	614.125	394.795	7.337234
8.75	669.921875	430.664	7.553688
9.	729.	468.643	7.769462
9.25	791.453125	508.791	7.984344
9.5	875.375	562.741	8.257263
9.75	926.859375	595.837	8.415541
10.	1000.	642.857	8.631103
10.25	1076.890625	692.287	8.845085
10.5	1157.625	744.187	9.061309
10.75	1242.296875	798.619	9.279308
11.	1452.	933.428	9.771484

Before proceeding to consider the bodies of Shafts, subject to *lateral stress*, I shall inquire into the Strength of *Journals* of Shafts subject to torsion.

CHAP. III.

SECTION I.

(19) *Of the Strength of Journals, when the Stress arises from Torsion or Twisting, in addition to lateral Stress.*

In my inquiry into the strength and durability of the teeth of wheels, I have used what is called the *horses' power*, as a measure for the strain. I beg leave to refer the reader to what I have there said, (p. 130-134) in explanation of that term which I shall use here, in measuring the strain brought on shafts by torsion or twisting.

In this case of torsion, as well as that of lateral pressure, the law of proportionate strength *is as the cubes of the diameters* *.

For the sake of comparison, some Journals in actual use are arranged in the following Table.

It may be proper here to remark, that what I have to say respecting Journals, relates to those of cast iron, for although wrought iron will bear more lateral stress, as we have seen (No. 19), yet it is a fact, perhaps, not generally known, that wrought iron will not resist torsion equal to cast iron.

Explanation of the Table of Journals.

The Table contains first the diameters of the Journal subject to torsion, as really at work.

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* See Gregory's Mechanics, vol. i. page 126.

Secondly, those Journals all reduced to one denomination, which is done as follows:

1st. By supposing them all reduced to 10 revolutions per minute, *the stress being in the inverse ratio of the velocity.* (See Emerson, page 179) This is done by the following rule; As the number of revolutions per minute which the shaft is actually making is to 10, so is the present horses' power to the horses' power required.

Example. As 58 revolutions per min.
Is to 10 horses' power,
So is 32 horses' power
To 5.5, ans. horses' power.

2d. By supposing the revolutions thus reduced to 10, and the horses all reduced to 10, to find the proportionate size of Journals.

Rule.—As the horses' power to which

the Journals are equal, when reduced to 10 revolutions per minute, is to the cube of the present diameter, so is ten horses' power to the cube of the diameter required.

Let us take A in the Table again as an example.

As 5.5
Is to the cube of 7, viz. 421.875.
So is ten
To 767, the cube root of which is
9.153737, ans.

Column 1, contains letters to distinguish the cases and their description.
——— 2, The horses' power at which the resistance is valued.
——— 3, The diameter in inches of the Journal, as actually made.
——— 4, The revolutions per minute which they actually make.
——— 5, The horses' power to which the resistance would be

equal, if the shafts were all moving at 10 revolutions per minute.

Column 6, Diameters of gudgeons, if all reduced to one denomination, viz. 10 horses' power, at 10 revolutions per minute.

OBSERVATIONS ON THE TABLE.

It is to be observed that when these Journals are all reduced to one denomination their values appear very different.

The case A gives . . 9.1
————G only . . 3.0

But there are good reasons for this difference: A has not only torsion to resist, but also to carry a very heavy fly wheel, and it is prudent in such cases not merely to make an allowance for the weight properly balanced, but also for any inaccuracy which may occasion swagging, which greatly adds to the stress. G was found from experience to be at the very minimum, and it had hardly any other resistance but what arose from torsion.

It is further observable that the value for 10 horses in the smaller Engines is much less than in the larger. For this difference what I have said respecting the weight of the fly in a great measure accounts. And not only to the heavier fly to be considered but the greater danger of accidents from a large fly than from one that is of a smaller diameter.

SECTION II.

Of proportioning Journals to the Stress which they have to sustain.

(20) The stress any journal has to sustain being as the horses' power to which the resistance is equal *directly*, and the number of revolutions which the shaft makes *inversely*, it follows: That a re-

sistance for example of 32 horses' power on a Journal making 50 revolutions per minute has the very same stress with a resistance of 16 horses' power on another journal making 25 revolutions per minute.

50 divided by 32 is equal to 25 divided by 16, each of which gives a quotient of 0.64. Therefore in all cases when the horses' power divided by the revolutions per minute produces the same quotient the stress is the same.

Thus a resistance equal to 50 horses' power making 50 revolutions per minute produces the very same stress as 10 horses' power making 10 revolutions per minute.

Having therefore fixed on any Journal which has been found sufficiently strong, we may make any other to have the same strength in proportion to the resistance which it has to overcome in the following manner.

If it so happen that the horses' power, and the revolutions per minute be the *same number*. For instance, 50 horses' power making 50 revolutions, $50 \div 50 = 1$; then the cube of the diameter of the Journal will be a multiplier, by which to find the cube of the diameter of the required Journal.

But in case the horses' power and the revolutions per minute are *different numbers*, then you must suppose them both the same, and calculate (by the rules formerly laid down (No. 19) what, in that case, would be the proportionate diameter of the Journal—The cube of this diameter will be a multiplier, the same as mentioned above.

Having found the multiplier, to find the diameter of the required Journal.

Divide the horses' power by the revolutions per minute.

Multiply the quotient by the *multiplier*, the cube root of the product will give the diameter of the Journal required.

EXAMPLE I.

In the 1st table of Journals at D, where there is an engine of 50 horses' power turning a shaft, at the rate of 50 revolutions per minute.

Divide the power by the revolutions, that is 50, divided by 50, is equal to 1; the diameter of the Journal is 7½ inches; the cube of this, is 420, which multiplied by 1, produces 420.

In this case it happens that the horses' power, and the revolutions per minute, are *the same number*, therefore we with little trouble find the multiplier.

EXAMPLE II.

Let us take E, in the table Here the horses' power is 12, and the revolutions per minute, 48.

Now let us suppose both numbers the same, that is 12 horses' power, and 12 revolutions.

Here it is evident that there will be four times the stress, brought on the Journal. Its actual diameter was 4 inches.

Then the cube of 4 is 64,

64 multiplied by 4 is 256 inches,

The cube root of which is 6.34,

which is the diameter, which the Journal ought to have, to be in proportion to the velocity.

The cube of 6.34 is 254.84,

which is the multiplier required.

It is to be observed, that in the case E, a much smaller steam engine is employed than in D, and, therefore, for reasons already given (No. 19.) has less stress brought upon the Journal. This accounts for the multiplier being less.

I shall now give an example of the application of a *multiplier ;* let us take that found in the case D, viz. 420, and see which size of Journal it would give in the case E, which is an engine of 12 horses' power and Journal making 48 revolutions per minute.

12 divided by 48, equal to 25, then multiplied by 420, gives a quotient of 105, that is, the strength of the Journals must be as 420 to 105; but the cube root of 420 is 7½, and the cube root of 105 is 4¾, which points out the Journals 7½ and 4¾ are proportioned to their respective engines. In like manner the following table is calculated; the multiplier being 420, the Journals in

column 7th, are proportional to the case C.

Description of the Table of Journals, proportionate to D, *having* 420 *as a multiplier.*

Column 1	contains letters to distinguish the cases, and which refer to the same Steam Engine as those in the former Table.
Column 2	The horses' power.
Column 3	The revolutions of the Journal per minute.
Column 4	The product of the Horses' power divided by the revolutions of the Shaft.
Column 5	Contains the proportionate strain on each Journal, represented in whole numbers, which are found by multiplying the product in column 4, by 420, as a multiplier.

Column 6 Contains the cube roots of the numbers, in column 5, that is, the diameters of Journals proportionte to that actually used in the case C.

Column 7 Diameters of Journals, as really executed in the several Steam Engines, referred to in the Table.

II. TABLE OF JOURNALS

Proportionate to D, *having* 420 *as a multiplier.*

1	2	3	4	5	6	7
	Numbers of horses power.	Revolutions of Journal p. min.	Product of power divided by the rev. of the Journal.	Proportionate strain on Journal.	Proportionate Diam. of Journals	Diam. of Journals from observation.
A	82	58	0.55	231	6.13	7½
B	32	19	1.67	701	8.88	9½
D	50	50	1.0	420	7.48	7½
E	12	48	0.25	105	471	4
F	9	55	0.16	67	4.06	4

OBSERVATIONS,

(21) I have already observed (No. 19) that, besides torsion, the journals of fly-wheel shafts have considerable lateral, and other stress, arising from the weight and swagging of their fly wheels, and therefore, they ought to be made stronger than shafts, in other situations. The multiplier 420, therefore, which we have used in the table, would give diameters too great, for some other parts of machinery. A Journal, for instance, subject to torsion, immediately connected with a water wheel, has from the weight of the wheel and other causes, considerable lateral stress; but not so much as that of a Steam Engine. The Journal may therefore, be considerably smaller than would be required for a Steam Engine fly wheel shaft, subject to the same degree of Torsion.

Again a secondary shaft driven from a Steam Engine water wheel, or horse gin, by means of wheels, has in general very little lateral stress, compared with the two cases just stated; and may therefore have a Journal, smaller than either, when the degree of torsion is the same.

For these reasons the three following multipliers will probably approach near the truth; for Journals of Steam Engine fly wheel shafts (where the power is moderate).

————————————	400
Journals in immediate connection with water wheels,* or other heavy work.	200
Journals for the ordinary kind of internal Millwork	100

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[*] The reader will please to observe, that, when I use the word Journal here, I suppose it subject to torsion. Where there is lateral pressure only, and no torsion, I would use the word gudgeon.

I shall give an Example of each of these multipliers.

EXAMPLE I.

Suppose B, in the IId Table, 1.67 multiplied by 400, is 668, the cube root of which, is 8.7416 inches, diameter of Journal.

EXAMPLE II.

Suppose G, in the Ist Table, then $7 \div 11\frac{1}{2} = 0.6 \quad 0.6 \times 200 = 120.$ The cube root of which is 4.932 inches, diameter of Journal.

EXAMPLE III.

Suppose G again, then $0.6 \times 100 = 60$, the cube root of which, is 3.914 inches, diameter of the Journal.

I have taken G for an example, in the two last cases, as it was in practice found to be at the minimum, or least extreme; were a multiplier taken from its real diameter, it would be rather less than 30.

For 0.6 × 30 = 18, the cube root of which, is 2.62, now the diameter from observation was 2$\frac{9}{16}$ inches, or 2.56.

SECTION III.

(22) When the diameter of a Journal, and its revolutions per minute, are given, in order *to find the horses' power* to which it is equal; we must invert the preceding operation, and convert the *multiplier* into a *divisor*.

RULE.

Cube the diameter of the Journal, divide the cube by the *divisor*. The quotient multiplied by the revolutions per minute, gives the horses' power, to which the Journal is equal.

EXAMPLE I. F *in the Table.*

Suppose the Journal of a Steam Engine to be 4 inches diameter, making 55 revolutions per minute, then we shall use 400 as a divisor.

=(0,16)
55

The cube of 4 is 64, divided by 400 equal to 0.16, then this quotient (0.16) multiplied by 55, gives 8.8 the horses' power, to which the Journal is equal.

EXAMPLE II.

Suppose again, the same size of a Journal, connected with heavy machinery, then we must use 200 as a divisor.

The cube of 4 = 64 ÷ 200 = .32 × 55 = 17.6 horses' power.

EXAMPLE III.

We shall take the same Journal for internal work, of the ordinary kind, and use 100 as a divisor.

The cube of $4 = 64 \div 100 = .64 \times 55 = 35.2$ horses' power.

In this way the three following tables, are calculated, from the three divisors 400, 200 and 100, already described. The first embraces the ordinary cases of Steam Engines, the second, those of Water Wheels and heavy machinery, and the third, internal work, of the ordinary kind.

Description of the three following Tables.

Column 1 contains the diameters of the Journals in inches.

The rest of the columns contain the horses' power, to which the gudgeons are equal at various velocities, from 1 revolution per minute to 60.

Use of the Tables.

EXAMPLE.

How many horses' power is a Journal of 2 inches diameter equal to, at 50 revolutions per minute, in each of the three Tables.

On a line with 2 inches diameter, in the first column of each table, the answers will be found in the 7th column, viz.

A Journal, 2 inches diameter, making 50 revolutions per minute.

For fly wheel shaft of Steam Engine 1 horse's power.

For heavy internal work 2 horses' power, For internal work of the usual kind 4 horses' power.

N.B. As column 2 contains the horses' power, to which the Journal is equal, at 1 revolution per minute, it is easy from it, to find the horses' power equal to any other number of revolutions by multiplication.

CHAP IV.

OF THE BODIES OF SHAFTS.

SECTION I.

(23) From what is stated in the preceding part of this Essay, I hope the Millwright will be enabled to approach sufficiently near to the truth, for all practical purposes in proportioning Gudgeons and Journals to the stress which they have to sustain. Taking this for granted, it may be proper for us next to consider what relates to the *bodies* of shafts, or those parts which lie between the Gudgeons or Journals. In this part of our inquiry, we may derive assistance from the principles, which have been applied

by writers on mechanics, to the stress of timber and other materials.* The generality of writers on mechanics, however, as Dr. Young justly observes, in (vol. i. page 136) have confined their attention to *strength* alone, although there be other very important properties, which required their consideration. The most usual as well as the most important effect, produced by the application of force is *flexure* (p. 138, ibid.) *stiffness* therefore as well as *strength*, ought to be considered in determining the form, as well as the quantity, of materials, for any mechanical purpose, more particularly that of a shaft in Millwork, which in theory may be considered as an *inflexible* straight line. I beg the practical reader would attend to the distinction between *stiffness* and *strength*. Stiffness is that property, which resists *flexure* or *bending*. Strength, that which resists fracture or breaking. The consideration of their limits, may make this plainer. The limit of *stiffness* is *flexure*: The limit of strength is *fracture*. The *stiffness* of a beam follows laws very different from those which determine its *strength*, those laws I shall presently consider, and endeavour to show their application to practice, with regard to some cases of shafts.—But although those laws may throw considerable light on the subject, yet it must be confessed, that there are many cases in practice, in which it is very difficult, if not impossible, to apply them; for it is very often difficult to estimate, what may be the amount of the lateral pressure on shafts, arising not only from their own weight, and that of the wheels upon them, but also from the thrust, proceeding from the action of the toothed wheels, and other extraneous causes. In cases of this nature, where calculation fails, much must be done, by what Mr. Smeaton calls *feeling*,* which will direct the experienced Millwright, to make a due allowance for whatever accidental strain may be likely to occur.

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* Few operative mechanics have a distinct notion of the difference between *strength* and *stiffness*.

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* Smeaton's Account of Edistone Lighthouse, page 136.

I shall now proceed, to state and apply some of the laws, respecting stiffness and strength, with regard to force, applied transversely.

Of lateral Stiffness and lateral Strength.

(24) "The *stiffness* of any substance, is measured by the force required, to cause it to recede, through a given space, in the direction of the force," (Young's Nat. Phil. vol. i. p. 139). Its transverse *strength* is measured by the pressure, required to produce its fracture, or, in other words, to break it.

PROPOSITION II.

(25) *Any beams of equal length, have their lateral Stiffness, as the breadth, and cube of the depth* (Young's Nat. Phil. vol. i. p. 139) *and have their lateral Strength,*

as the breadth and square of the depth[*] (Gregory, vol. i. p. 106, Emerson, page 94, cor. 2).

Thus, if a square beam measure twice as much, on the side as another of equal length, it would be sixteen times as stiff. In other words, it will sustain sixteen times the weight, without bending.

But, if a square beam be twice as much on the side as another, both being the same length, it will be only eight times stronger.

Hence we see, that when beams or shafts are of equal lengths, their *stiffness*, by any increase of thickness, increases in a higher proportion than their *strength*.

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[*] That is, as the cube of the side of a square beam, and in general the lateral strength of any beams whose sections are similar, as the cube of the similar sides or diameters of the sections.

EXAMPLE I.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative *stiffness?*

The cube of 4, is 64,

$64 \times 4 = 256$,

The cube of 5, is 125,

$125 \times 5 = 625$,

That is, the shaft of five inches is nearly two and a half times stiffer, than that of four; in other words, it would require nearly two and a half times the weight, to bend it.

EXAMPLE II.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative strength?

The cube of 4, is 64,

The cube of 5, is 125,

That is, the five inches shaft, is nearly twice as strong, as that of four inches; in other words, it would require nearly double the weight, to break it.

PROPOSITION III.

(26) *Any beams of different lengths, have their stiffness directly as the breadth, and the cube of the depth, and inversely as the cube of the length, and have their strength directly as the breadth, and as the square of the depth, and inversely as the length*** (Young, vol. ii. p. 50, Gregory, vol. i. p. 106).

Thus, if a beam be twice as long as another, of the same breadth and depth, it will have only one eighth of the stiffness, while it will have one half of the strength. Hence the *stiffness* of shafts or beams by any increase of their length, decrease in a much higher proportion than that of their *strength.*

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* "This is not strictly true in practice, for some experiments appear to show, that the strength is diminished, in a proportion somewhat greater than that in which the length is increased" (Young's Nat. Phil. vol. ii. page 147).

EXAMPLE I.

Suppose a beam or shaft, four feet long and four inches square throughout, and another eight feet long and 7 inches square; what is their comparative stiffness?

The cube of 4 feet, is 64,

The cube of 8 feet, is 512,

512 divided by 64, is equal to 8, that is, when we double the length, we decrease the stiffness eight times.

The cube of 4 inches is 64, which multiplied by 4 is equal to 256, a number representing the stiffness of the four inch shaft.

The cube of 7 inches is 343, multiplied by 7 is equal to 2401, divided by 8 is equal to 300.1, then as 256 is to 300.1, so is the stiffness of the shaft of four inches to that of a shaft of seven inches.

EXAMPLE II.

Suppose a beam or shaft, four feet long and three inches square, and another eight feet long and seven inches square, what is their comparative strength.

The cube of 4 is 64, which represents the strength of the four inch shaft.

The cube of 7 is 343; but the shaft being of double length, we must half this sum, to find the number representing its strength, viz. 343 ÷ 2 = 171.5 divided by 100, that is, as sixty-four is to a hundred and seventy-one and a half, so is the strength of the short shaft, to that of the long. Thus the shaft of seven inches, eight feet long, has nearly two and six tenth times the strength, as the four inch four feet long.

PROPOSITION IV.

(27) *Supposing a tube, indefinitely thin, to be expanded into a similar tube of a greater diameter, but of equal lengths, the quantity of matter remaining the same, the* STIFFNESS *will be increased, in the** *ratio of the square of the diameter, and the* STRENGTH *in the ratio of the diameter.*

Thus, if the one tube be double the diameter of the other, it will have four times its *stiffness*, but only double the strength.

Hence, hollow cylinders of equal lengths, by any increase of diameter, increase in *stiffness*, in a much higher proportion than in *strength*.

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* Ratio, that is proportion.

EXAMPLE I

Suppose two thin narrow cylindrical cast-iron shafts, of equal lengths and weights, the one one foot diameter, and the other three feet diameter, required their comparative *stiffness.*

The square of one is one,
The square of three is nine,

that is, the shaft of three feet diameter, is nine times stiffer than that of one foot.

EXAMPLE II.

Suppose the same shafts, as in example first, required their comparative strength?

Diameter one foot,
Diameter three feet,

that is, the three feet shaft is just three times stronger, than that of one foot diameter.

(28) In these examples, we have supposed the weight of the shafts equal, that is the area of their ends to be equal, but the strength of any of them would be increased in proportion to their weight, or the areas of their ends and diameters, conjointly (Gregory, vol. i. p. 109.)

Thus suppose two Shafts of equal length and diameter, the one double the weight of the other, it will be double the strength.

(29) Professor Robison justly observes, " That this property of hollow tubes is accompanied also with greater stiffness, and the superiority in strength and stiffness is so much the greater, as the surrounding shell is thinner in proportion to its diameter. Here we see the admirable wisdom of the Author of nature in forming the bones of animal limbs hollow. The bones of the arms and legs have to perform the office of levers, and are thus opposed to very great transverse strains.

By this form they become incomparably stronger and stiffer, and give more room for the insertion of muscles, while they are lighter and therefore more agile; and the same wisdom has made use of this hollow for other valuable purposes of the animal economy. In like manner the quills in the wings of birds acquire by their thinness the very great strength which is necessary; while they are so light as to give sufficient buoyancy to the animal, in the rare medium in which it must live and fly about. The stalks of many plants, such as all the grasses, and many reeds, are in like manner hollow, and thus possess an extraordinary strength. (Ency. Brit. article strength.)

Long before this eminent Philosopher, the celebrated Galileo made similar observations, and goes on to say that "if a wheat straw, which supports an ear that is heavier than the whole stalk, were made of the same quantity of matter but solid, it would bend or break with far greater ease than it now does. And with the same reason art has observed and experience confirmed, that a hollow cane or tube of wood or metal, is much stronger and more firm than if, while it continued of the same weight and length, it were solid, as it would then of consequence be not so thick.

It may be proper now to consider the effects called *Stress*, which are produced on beams or shafts lying horizontally by weights or pressures brought on various parts of them.

SECTION II.

Of Lateral Stress.

(30) The *Stress* or *Strain* are terms used to express the force which is excited in any body tending to break it. The meaning of the term *Stress* may perhaps be more

clearly understoood by contrasting it with the term *Strength.*

Strength, as we have already observed, is the property which *resists fracture.*

Stress is that which has the tendency TO PRODUCE *fracture*; and *Lateral Stress*, is that particular application of it, which has the tendency to break a body *across.*

PROPOSITION V.

(31) *The Stress on a beam arising from one weight hung upon it, is proportional to the rectangle of the parts of the beam, and is greatest when the load is laid on the middle of the beam.* (Ency. Brit. art. Roof, § 19.)

What is meant by the expression *rectangle of the parts*, is the product of the parts multiplied into each other. Thus for example; if a beam be ten feet long, and the weight hung two feet from one end, the parts are 2 and 8, which multiplied together, is equal to 16; but supposing the weight were hung in the middle, the parts are 5 and 5, which multiplied together would produce 25.

(32) The ends of beams having the whole weight to support; the end which is nearest the weight, has to support the greatest proportion of it, in the inverse proportion of the distance of the weight from the end. This will be easily understood from the properties of the lever. For, suppose the beam instead of being supported by two props or walls, as in fig. 9, No. 1, to be hung at each end from a rope, as in fig. 9, No. 2, it is plain that the beam would receive the same support, and suffer the same Stress, as if lying on props or walls: now suppose the weights A and B, to balance the weight W, A and B, taken together, must be equal to W, but A must be greater than B in proportion as W is near to it.

(33) Hence when any beams or shafts

are loaded exactly in the middle, each of the ends of the beams or gudgeons of the shafts has half the weight to support, and when the weight is nearer one end, the end or gudgeon to which it is nearest, has the stress in the inverse proportion of the distance. In this last case therefore the one gudgeon should be smaller than the other.

(34) "We may always consider the weight which is uniformly diffused over any part of a beam as united in the middle of that part, and if the load is not uniformly diffused, we may suppose it united at its centre of gravity." (En. Brit. article Roof. § 20.)

(35) It is evidently of importance that a beam or shaft should in every part be able to resist the strain excited in that part. "It should therefore be equally strong, because the piece will nevertheless break where it is not stronger throughout, and it is useless to make it stronger

(relatively to its strain) in any part, or it will nevertheless equally fail in the part that is too weak." (Ibid.)

(36) From what we have said respecting lateral stress, it is evident that when a beam lying between two props is loaded at some intermediate part, that part has to sustain more stress than the rest. In order to resist this strain therefore, and to render the beam equally strong throughout, it should have its section enlarged at the place of greatest stress, and hence shafts subject to lateral stress, should swell in the middle, and it will be found that when each section is made proportional to the stress it has to sustain that the sides of the shaft will form curves.

(37) When the transverse sections of a beam are all similar, such as circles, squares, or polygons, and the weight is laid on one place, in order to make it equally strong throughout its length, the curve of the sides of the beam becomes

what Mathematicians call a cubical parabola. (Ency. Brit. Strength of Materials, 87.) But when the weight is uniformly diffused all over the beam, the sides of the beam must be a different parabolic curve called a semicubical parabola.*

We come now to examine some of the laws respecting Twisting, or Torsion.

SECTION III.

Of Torsion.

PROPOSITION VI.

(38) *In general the strength of a cylinder or solid axle by which it resists being wrenched asunder by twisting is as the cube of its diameter.* (Enc. Brit. art. Strength of Materials, 123.)

Thus if a solid cylinder be double the diameter of another, it would require eight times the force to wrench it asunder.

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* "The parabola is a conic section, arising from a cone being cut by a plane parallel to one of its sides, or parallel to a plane that touches one side of the cone."

Of Hollow Axles.

(39) Hollow Axles are stronger to resist twisting than solid ones containing the same quantity of matter. For if a hole be bored out of an axle of half its diameter, this reduces its *weight one fourth* (because circles are to one another as the squares of their diameters), but the strength of solid cylinders being as the cubes of their diameters the part taken out by boring had only the eighth part of the strength of the whole cylinder and therefore when taken out would reduce the *strength* of the whole *one eighth.*

Thus let the external diameter of the Hollow Axle be five inches, and that of the hollow part of it four inches, then the diameter of another cylinder made solid, having the same quantity of metal with the tube, is three inches.

For 5 multiplied by 5, is equal to 25.
4 multiplied by 4, is equal to 16.
Difference 9.

The square root of 9 is 3. The strength of the solid cylinder of five inches diameter may be expressed by the cube of 5, or 125. Of this the internal part four inches diameter exerts 64, that is, the cube of 4; therefore the strength of the tube is 64, subtracted from 125, is equal to 61, but the strength of the solid axle of the same quantity of matter, and three inches diameter is expressed by the cube of 3 or 27, which is not half of that of the tube. (En. Brit. art. Strength of Materials, 124.)

(40) The superiority of strength of hollow tubes over solid cylinders is much greater in resisting torsion than transverse or lateral stress. We have seen above, that the strength to resist torsion of the tube was to that of the cylinder as sixty-one is to twenty-seven; but Professor

Robison estimates, that their strength to resist transverse strain is only as sixty-one is to thirty-two and a half nearly—and if we calculate according to Mr. Gregory's corollary, vol. i. page 109, (see No. 28 of this Essay) the result will be still more in favour of strength to resist torsion, for by the last mode of calculation the tube would be to the cylinder only as forty-five is to thirty-six; but the Professor's mode of calculation, though less simple, is probably more accurate than that above alluded to.

(41) Professor Robison mentions (in En. Brit. Article Strength of Materials, § 128) that "when the matter of the axle is of the most simple texture, such as that of metals, we do not conceive that the length of the axle has any influence on the fracture. It is otherwise if it be a fibrous texture, like timber; the fibres are bent before breaking, being twisted into spirals like a cork screw. The length of the axle has somewhat of the influence

of a lever in this case, and it is easier wrenched asunder if long.—Accordingly we have found it so; but we have not been able to reduce this influence to calculation."

Having thus stated these laws, as far as seems to be necessary for the purpose of our present inquiry, it may be proper next to endeavour to apply them more particularly to practice, with regard to the proportion of shafts.

SECTION IV.

Of the Proportion of Shafts.

(42) It was already observed (No. 10) that the gudgeons or journals having to support the whole stress of the shafts to which they belong, their diameters being determined, serve to guide in determining the proportions of the whole shaft.

They are subject to wear, which the body seldom is. They ought, therefore, to be sufficiently large to allow for that wear. It frequently happens, that a shaft has no lateral pressure excepting that which arises from its own weight, for instance, in a line of coupled horizontal shafts conveying motion to a distance. In the case of vertical shafts also there is often little or no lateral stress. In such cases, when solid cast iron shafts are of moderate lengths, it is found from experience, that making them square of the same size throughout, between the journals, and the measuring a little more on the side than their diameters, gives them sufficient stiffness. Even where there is considerable lateral stress when the shafts are but short, making them in this manner is found to give sufficient stiffness. This square form, in many cases, affords great convenience for hanging or fixing wheels, pulleys, &c. upon them.

(43) The gudgeons of water wheels

are often so near the wheel, that the stress is, in a great measure, taken off the shaft. Hence some water wheels are made without shafts, the gudgeons being fixed to the arms at each side of the wheel. The sole of the wheel, in these cases, may be *considered as a large hollow axle.*

Cast Iron Shafts.

(44) 1st. Let us now suppose a square cast iron shaft required to be made eight feet long, with gudgeons four inches diameter, having considerable lateral stress in the middle—were the shaft of equal size throughout, it would evidently be weakest in the middle. Its section, in that place, ought therefore to be increased. We have already seen (No. 42) that square shafts, when short, are sufficiently stiff, if made as large as the gudgeons; this shaft may therefore be four inches square at the end. Supposing the section in the middle enlarged to five

inches, swelling with a regular curve (No. 37,) from each end (by Prop. 2d.) Then

The cube of 4 is 64
The cube of 5 is 125

that is nearly double.—Therefore, the *strength* of shafts being inversely as their length (Prop. 3d) the section as thus increased, would make the eight feet shaft nearly as strong as a shaft subject to the same stress, only four feet long and four inches in the middle.

Let us next see what would be the *stiffness* of this shaft, as thus swelled in the middle, to five inches. Then (by Proposition 2d), the cube of 4 is 64, which, multiplied by 4 is equal to 256. The cube of 5 is 125, multiplied by 5, is equal to 625; therefore, the stiffness being inversely as the cube of the lengths—

The cube root of 256 is 6.3
The cube root of 625 is 8.5

Therefore, the section, as thus increased, would make the eight feet shaft about as stiff as one of six feet, subject to the same stress, and four inches throughout.

(45) 2d. Let us next suppose the point of greatest lateral pressure to be two feet from one end.

From the properties of the lever, the gudgeon next the point of greatest pressure has three-fourths of the whole to sustain.

Now the cube of the diameter of the gudgeon is 64 multiplied by 2, is equal to 128, which represents the strength of both journals taken together.

3-4ths of 128 is equal to 96.

The cube root of 96 is 4.578, the diameter which the largest gudgeon ought to have—

1-4th of 128 is 32.

The cube root of 32 is equal to 3.174,

the diameter which the smallest gudgeon ought to have, or that furthest from the point of pressure.

Having thus obtained the diameters of the two gudgeons, each end of the shaft may be made square, equal to its respective gudgeon (No. 42).

The stress on the shaft being as the rectangle of the parts, (Prop. 5); in this case it is less than the former, as twelve is to sixteen—for four feet multiplied by four feet is equal to 16, which expresses the stress when in the middle—6 feet multiplied by two feet is equal to 12, which expresses the stress when two feet from one end.

This shaft would therefore be as *strong* as the former, if its greatest section were reduced to 4.54 :

For the cube of 5 is 125, as 16 : 12 : : 125 93.75, the cube root of which is 4.54.

It would be as *stiff* as the former, if its greatest section were reduced to about 4.65:
For the cube of 5 is equal to 125; multiplied by 5 is equal to 625, which expresses the stiffness of the former shaft, as 16 : 12 : : 625 : 468.75, which expresses the required stiffness; and the cube of 4.65, is equal to 100.544625, which, multiplied by 4.654, is equal to 467.934, which is nearly equal to the required stiffness.

By examining shafts in this way, the mill-wright, according to the nature of the case, will be enabled to judge how much they should swell at the place of *greatest stress.*

Wooden Shafts.

(46) 3d. Suppose a cast iron shaft five inches square at the point of greatest lateral stress—required the size

it should be when made of oak to have the same strength.

The cube of 5 is 125.

Cast iron is four times stronger than oak—(See Banks on Powers of Machines, &c. p. 94.) therefore 125 + 4 = 500.

The cube root of 500 is 7.93 inches.

(47) 4th. Required the size of a fir shaft to have the same strength as the cast iron.

The cube of 5 is 125.

Cast iron—5.5 times stronger than fir (ibid.) therefore, 125 + 5.5 = 687.5; the cube root of 687.5 is equal to 8.82, answer.

(48) In the same manner we might examine various kinds of cylindrical shafts, but I presume what I have already said will suffice. In order to shew, however, that considerable allowance should be made for accidental stress, I

may mention the following fact which lately occurred.

(49) The hollow shaft of a water-wheel, in consequence of one of the gudgeons getting loose, broke quite through near one end, although it was 16 inches diameter, and two inches thick in the shell—(See No. 13, 1st Table of Gudgeons). The gudgeons of this shaft were not secured by flanches.

(50) Hollow cylindrical shafts are often made of equal size throughout, in order to get large flanches, the better to secure the gudgeon.

Shafts subject to Torsion.

(51) We saw (No. 41) that the length of a cast iron shaft has no influence on its resistance to Torsion—and I have already said all that seems to be necessary respecting them. The case is, how-

ever, different with regard to wooden shafts; but we are yet in want of facts to reduce the influence which their length may have, to calculation. As it may give some idea of this influence, I beg leave to state one fact which came recently under my observation.

A shaft about 15 feet long, made of fir, had cast iron journals $2\frac{9}{16}$ diameter—(See G. Table of Journals) one of the journals broke from fair stress after working about 16 years; for a considerable time before it broke, the resistance was equal to 7 horses' power, making $11\frac{1}{2}$ revolutions per minute. It gradually wore until it broke; when that happened, the shaft seemed strained to the utmost, so that it might be reckoned as just equal in strength to the gudgeon; the shaft was $9\frac{3}{4}$ inches square.

The cube of $9\frac{3}{4}$ is 926,

926 divided by 5.5 (No. 47) is 16.8;

Cube root of 16.8 is 2.6. That is, the fir shaft would be equal to resist the same *lateral stress* as a square cast iron shaft 2.6 on the side.

The cross-tailed gudgeons of wooden shafts commonly require them to be made sufficiently large to withstand the stress which is brought upon them; often indeed, formerly, they were much weakened by mortices cut through them for inserting the arms of wheels; this practice is now, however, almost entirely abandoned.

I hope some tables which I have annexed respecting the strength of several substances, will be acceptable to the reader; the table of squares and cubes; as also the square roots and cube roots, of all numbers from one to 1000, which I have taken the liberty to extract from Dr Hutton's "Course of Mathematics," will be found very useful on many occasions.

Number	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1·0000000	1·000000
2	4	8	1·4142136	1·259921
3	9	27	1·7320508	1·442250
4	16	64	2·0000000	1·587401
5	25	125	2·2360680	1·709976
6	36	216	2·4494897	1·817121
7	49	343	2·6457513	1·912933
8	64	512	2·8284271	2·000000
9	81	729	3·0000000	2·080084
10	100	1000	3·1622777	2·154435
11	121	1331	3·3166248	2·223980
12	144	1728	3·4641016	2·289428
13	169	2197	3·6055513	2·351335
14	196	2744	3·7416574	2·410142
15	225	3375	3·8729833	2·466212
16	256	4096	4·0000000	2·519842
17	289	4913	4·1231056	2·571282
18	324	5832	4·2426407	2·620741
19	361	6859	4·3588989	2·668402
20	400	8000	4·4721360	2·714418
21	441	9261	4·5825757	2·758923
22	484	10648	4·6904158	2·802039
23	529	12167	4·7958315	2·843867
24	576	13824	4·8989795	2·884499
25	625	15625	5·0000000	2·924018
26	676	17576	5·0990195	2·962496
27	729	19683	5·1961524	3·000000
28	784	21952	5·2915026	3·036589
29	841	24389	5·3851648	3·072317
30	900	27000	5·4772256	3·107232
31	961	29791	5·5677644	3.141381
32	1024	32768	5·6568542	3·174802
33	1089	35937	5·7445626	3·207534
34	1156	39304	5·8309519	3·239612
35	1225	42875	5·9160798	3·271066
36	1296	46656	6·0000000	3·301927
37	1369	50653	6·0827625	3·332222
38	1444	54872	6·1644140	3·361975
39	1521	59319	6·2449980	3·391211
40	1600	64000	6·3245553	3·419952
41	1681	68921	6·4031242	3·448217
42	1764	74088	6·4807407	3·476027
43	1849	79507	6·5574385	3·503398
44	1936	85184	6·6332496	3·530348
45	2025	91125	6·7082039	3·556893

Number	Square.	Cube.	Square Root.	Cube Root.
46	2116	97336	6·7823300	3·583048
47	2209	103823	6·8556546	3·608826
48	2304	110592	6·9282032	3·634241
49	2401	117649	7·0000000	3·659306
50	2500	125000	7·0710678	3·684031
51	2601	132651	7·1414284	3·708430
52	2704	140608	7·2111026	3·732511
53	2809	148877	7·2801099	3·756286
54	2916	157464	7·3484692	3·779763
55	3025	166375	7·4161985	3·802953
56	3136	175616	7·4833148	3·825862
57	3249	185193	7·5498344	3·848501
58	3364	195112	7·6157731	3·870877
59	3481	205379	7·6811457	3·892996
60	3600	216000	7·7459667	3·914867
61	3721	226981	7·8102497	3·936497
62	3844	238328	7·8740079	3·957892
63	3969	250047	7·9372539	3·979057
64	4096	262144	8·0000000	4·000000
65	4225	274625	8·0622577	4·020726
66	4356	287496	8·1240384	4·041240
67	4489	300763	8·1853528	4·061548
68	4624	314432	8·2462113	4·081656
69	4761	328509	8·3066239	4·101566
70	4900	343000	8·3666003	4·121285
71	5041	357911	8·4261498	4·140818
72	5184	373248	8·4852814	4·160168
73	5329	389017	8·5440037	4·179339
74	5476	405224	8·6023253	4·198336
75	5625	421875	8·6602540	4·217163
76	5776	438976	8·7177979	4·235824
77	5929	456533	8·7749644	4·254321
78	6084	474552	8·8317609	4·272659
79	6241	493039	8·8881944	4·290841
80	6400	512000	8·9442719	4·308870
81	6561	531441	9·0000000	4·326749
82	6724	551368	9·0553851	4·344481
83	6889	571787	9·1104336	4·362071
84	7056	592704	9·1651514	4·379519
85	7225	614125	9·2195445	4·396830
86	7396	636056	9·2736185	4·414005
87	7569	658503	9·3273791	4·431047
88	7744	681472	9·3808315	4·447960
89	7921	704969	9·4339811	4·464745
90	8100	729000	9·4868330	4·481405

Number	Square.	Cube.	Square Root.	Cube Root.
91	8281	753571	9·5393920	4·497942
92	8464	778688	9·5916630	4·514357
93	8649	804357	9·6436508	4·530655
94	8836	830584	9·6953597	4·546836
95	9025	857375	9·7467943	4·562903
96	9216	884736	9·7979590	4·578857
97	9409	912673	9·8488578	4·594701
98	9604	941192	9·8994949	4·610436
99	9801	970299	9·9498744	4·626065
100	10000	1000000	10·0000000	4·641589
101	10201	1030301	10·0498756	4·657010
102	10404	1061208	10·0995049	4·672330
103	10609	1092727	10·1488916	4·687548
104	10816	1124864	10·1980390	4·702669
105	11025	1157625	10·2469508	4·717694
106	11236	1191016	10·2956301	4·732624
107	11449	1225043	10·3440804	4·747459
108	11664	1259712	10·3923048	4·762203
109	11881	1295029	10·4403065	4·776856
110	12100	1331000	10·4880885	4·791420
111	12321	1367631	10·5356538	4·805896
112	12544	1404928	10·5830052	4·820284
113	12769	1442897	10·6301458	4·834588
114	12996	1481544	10·6770783	4·848808
115	13225	1520875	10·7238053	4·862944
116	13456	1560896	10·7703296	4·876999
117	13689	1601613	10·8166538	4·890973
118	13924	1643032	10·8627805	4·904868
119	14161	1685159	10·9087121	4·918685
120	14400	1728000	10·9544512	4·932424
121	14641	1771561	11·0000000	4·946088
122	14884	1815848	11·0453610	4·959675
123	15129	1860867	11·0905365	4·973190
124	15376	1906624	11·1355287	4·986631
125	15625	1953125	11·1803399	5·000000
126	15876	2000376	11·2249722	5·013298
127	16129	2048383	11·2694277	5·026526
128	16384	2097152	11·3137085	5·039684
129	16641	2146689	11·3578167	5·052774
130	16900	2197000	11·4017543	5·065797
131	17161	2248091	11·4455231	5·078753
132	17424	2299968	11·4891253	5·091643
133	17689	2352637	11·5325626	5·104469
134	17956	2406104	11·5758369	5·117230
135	18225	2460375	11·6189500	5·129928

Number	Square.	Cube.	Square Root.	Cube Root.
136	18496	2515456	11·6619038	5·142563
137	18769	2571353	11·7046999	5·155137
138	19044	2628072	11·7473444	5·167649
139	19321	2685619	11·7898261	5·180101
140	19600	2744000	11·8321596	5·192494
141	19881	2803221	11·8743421	5·204828
142	20164	2863288	11·9163753	5·217103
143	20449	2924207	11·9582607	5·229321
144	20736	2985984	12·0000000	5·241482
145	21025	3048625	12·0415946	5·253588
146	21316	3112136	12·0830460	5·265637
147	21609	3176523	12·1243557	5·277632
148	21904	3241792	12·1655251	5·289572
149	22201	3307949	12·2065556	5·301459
150	22500	3375000	12·2474487	5·313293
151	22801	3442951	12·2882057	5·325074
152	23104	3511808	12·3288280	5·336803
153	23409	3581577	12·3693169	5·348481
154	23716	3652264	12·4096736	5·360108
155	24025	3723875	12·4498996	5·371685
156	24336	3796416	12·4899960	5·383213
157	24649	3869893	12·5299641	5·394690
158	24964	3944312	12·5698051	5·406120
159	25281	4019679	12·6095202	5·417501
160	25600	4096000	12·6491106	5·428835
161	25921	4173281	12·6885775	5·440122
162	26244	4251528	12·7279221	5·451362
163	26569	4330747	12·7671453	5·462556
164	26896	4410944	12·8062485	5·473703
165	27225	4492125	12·8452326	5·484806
166	27556	4574296	12·8840987	5·495865
167	27889	4657463	12·9228480	5·506879
168	28224	4741632	12·9614814	5·517848
169	28561	4826809	13·0000000	5·528775
170	28900	4913000	13·0384048	5·539658
171	29241	5000211	13·0766968	5·550499
172	29584	5088448	13·1148770	5·561298
173	29929	5177717	13·1529464	5·572054
174	30276	5268024	13·1909060	5·582770
175	30625	5359375	13·2287566	5·593445
176	30976	5451776	13·2664992	5·604079
177	31329	5545233	13·3041347	5·614673
178	31684	5639752	13·3416641	5·625226
179	32041	5735339	13·3790882	5·635741
180	32400	5832000	13·4164079	5·646216

Number	Square.	Cube.	Square Root.	Cube Root.
181	32761	5929741	13·4536240	5·656652
182	33124	6028568	13·4907376	5·667051
183	33489	6128487	13·5277493	5·677411
184	33856	6229504	13·5646600	5·687734
185	34225	6331625	13·6014705	5·698019
186	34596	6434856	13·6381817	5·708267
187	34969	6539203	13·6747943	5·718479
188	35344	6644672	13·7113092	5·728654
189	35721	6751269	13·7477271	5·738794
190	36100	6859000	13·7840488	5·748897
191	36481	6967871	13·8202750	5·758965
192	36864	7077888	13·8564065	5·768998
193	37249	7189057	13·8924440	5·778996
194	37636	7301384	13·9283883	5·788960
195	38025	7414875	13·9642400	5·798890
196	38416	7529536	13·0000000	5·808786
197	38809	7645373	14·0356688	5·818648
198	39204	7762392	14·0712473	5·828476
199	39601	7880599	14·1067360	5·838272
200	40000	8000000	14·1421356	5·848035
201	40401	8120601	14·1774469	5·857765
202	40804	8242408	14·2126704	5·867464
203	41209	8365427	14·2478068	5·877130
204	41616	8489664	14·2828569	5·886765
205	42025	8615125	14·3178211	5·896368
206	42436	8741816	14·3527001	5·905941
207	42849	8869743	14·3874946	5·915481
208	43264	8998912	14·4222051	5·924991
209	43681	9123329	14·4568323	5·934473
210	44100	9261000	14·4913767	5·943911
211	44521	9393931	14·5258390	5·953341
212	44944	9528128	14·5602198	5·962731
213	45369	9663597	14·5945195	5·972901
214	45796	9800344	14·6287388	5·981426
215	46225	9938375	14·6628783	5·990727
216	46656	10077696	14·6969385	6·000000
217	47089	10218313	14·7309199	6·009244
218	47524	10360282	14·7648231	6·018463
219	47961	10503459	14·7986486	6·027650
220	48400	10648000	14·8323970	6·036811
221	48841	10793861	14·8660687	6·045943
222	49284	10941048	14·8996644	6·055048
223	49729	11089567	14·9331845	6·064126
224	50176	11239424	14·9666295	6·073177
225	50625	11390625	15·0000000	6·082201

Number	Square.	Cube.	Square Root.	Cube Root.
226	51076	11543176	15·0332964	6·091199
227	51529	11697083	15·0665192	6·100170
228	51984	11852352	15·0996689	6·109115
229	52441	12008989	15·1327460	6·118032
230	52900	12167000	15·1657509	6·126925
231	53361	12326391	15·1986842	6·135792
232	53824	12487168	15·2315462	6·144634
233	54289	12649337	15·2643375	6·153449
234	54756	12812904	15·2970585	6·162239
235	55225	12977875	15·3297097	6·171005
236	55696	13144256	15·3622915	6·179747
237	56169	13312053	15·3948043	6·188463
238	56644	13481272	15·4272486	6·197154
239	57121	13651919	15·4596248	6·205821
240	57600	13824000	15·4919334	6·214464
241	58081	13997521	15·5241747	6·223083
242	58564	14172488	15·5563492	6·231678
243	59049	14348907	15·5884573	6·240251
244	59536	14526784	15·6204994	6·248800
245	60025	14706125	15·6524758	6·257324
246	60516	14886936	15·6843871	6·265826
247	61009	15069223	15·7162336	6·274304
248	61504	15252992	15·7480157	6·282760
249	62001	15438249	15·7797338	6·291194
250	62500	15625000	15·8113883	6·299604
251	63001	15813251	15·8429795	6·307992
252	63504	16003008	15·8745079	6·316359
253	64009	16194277	15·9059737	6·324704
254	64516	16387064	15·9373775	6·333025
255	65025	16581375	15·9687194	6·341325
256	65536	16777216	16·0000000	6·349602
257	66049	16974593	16·0312195	6·357859
258	66564	17173512	16·0623784	6·366095
259	67081	17373979	16·0934769	6·374310
260	67600	17576000	16·1245155	6·382504
261	68121	17779581	16·1554944	6·390676
262	68644	17984728	16·1864141	6·398827
263	69169	18191447	16·2172747	6·406958
264	69696	18399744	16·2480768	6·415068
265	70225	18609625	16·2788206	6·423157
266	70756	18821096	16·3095064	6·431226
267	71289	19034163	16·3401346	6·439275
268	71824	19248832	16·3707055	6·447305
269	72361	19465109	16·4012195	6·455314
270	72900	19683000	16·4316767	6·463304

Number	Square.	Cube.	Square Root.	Cube Root.
271	73441	19902511	16·4620776	6·471274
272	73984	20123648	16·4924225	6·479224
273	74529	20346417	16·5227116	6·487153
274	75076	20570824	16·5529454	6·495064
275	75625	20796875	16·5831240	6·502956
276	76176	21024576	16·6132477	6·510829
277	76729	21253933	16·6433170	6·518684
278	77284	21484952	16·6733320	6·526519
279	77841	21717639	16·7032931	6·534335
280	78400	21952000	16·7332005	6·542132
281	78961	22188041	16·7630546	6·549911
282	79524	22425768	16·7928556	6·557672
283	80089	22665187	16·8226038	6·565415
284	80656	22906304	16·8522995	6·573139
285	81225	23149125	16·8819430	6·580844
286	81796	23393656	16·9115345	6·588531
287	82369	23639903	16·9410743	6·596202
288	82944	23887872	16·9705627	6·603854
289	83521	24137569	17·0000000	6·611488
290	84100	24389000	17·0293864	6·619106
291	84681	24642171	17·0587221	6·626705
292	85264	24897088	17·0880075	6·634287
293	85849	25153757	17·1172428	6·641851
294	86436	25412184	17·1464282	6·649399
295	87025	25672375	17·1755640	6·656930
296	87616	25934336	17·2046505	6·664443
297	88209	26198073	17·2336879	6·671940
298	88804	26463592	17·2626765	6.679419
299	89401	26730899	17·2916165	6·686882
300	90000	27000000	17·3205081	6·694328
301	90601	27270901	17·3493516	6·701758
302	91204	27543608	17·3781472	6·709172
303	91809	27818127	17·4068952	6·716569
304	92416	28094464	17·4355958	6·723950
305	93025	28372625	17·4642492	6·731316
306	93636	28652616	17·4928557	6·738665
307	94249	28934443	17·5214155	6·745997
308	94864	29218112	17·5499288	6·753313
309	95481	29503629	17·5783958	6·760614
310	96100	29791000	17·6068169	6·767899
311	96721	30080231	17·6351921	6·775168
312	97344	30371328	17·6635217	6·782422
313	97969	30664297	17·6918060	6·789661
314	98596	30959144	17·7200451	6·796884
315	99225	31255875	17·7482393	6·804091

Number	Square.	Cube.	Square Root	Cube Root.
316	99856	31554496	17·7763888	6·811284
317	100489	31855013	17·8044938	6·818461
318	101124	32157432	17·8325545	6·825624
319	101761	32461759	17·8605711	6·832771
320	102400	32768000	17·8885438	6·839903
321	103041	33076161	17·9164729	6·847021
322	103684	33386248	17·9443584	6·854124
323	104329	33698267	17·9722008	6·861211
324	104976	34012224	18·0000000	6·868284
325	105625	34328125	18·0277564	6·875343
326	106276	34645976	18·0554701	6·882388
327	106929	34965783	18·0831413	6·889419
328	107584	35287552	18·1107703	6·896435
329	108241	35611289	18·1383571	6·903436
330	108900	35937000	18·1659021	6·910423
331	109561	36264691	18·1934054	6·917396
332	110224	36594368	18·2208672	6.924355
333	110889	36926037	18·2482876	6·931300
334	111556	37259704	18·2756669	6·938232
335	112225	37595375	18·3030052	6·945149
336	112896	37933056	18·3303028	6·952053
337	113569	38272753	18·3575598	6·958943
338	114244	38614472	18·3847763	6·965819
339	114921	38958219	18·4119526	6·972682
340	115600	39304000	18·4390889	6·979532
341	116281	39651821	18·4661853	6·986369
342	116964	40001688	18·4932420	6·993191
343	117649	40353607	18·5202592	7·000000
344	118336	40707584	18·5472370	7·006796
345	119025	41063625	18·5741756	7·013579
346	119716	41421736	18·6010752	7·020349
347	120409	41781923	18·6279360	7·027106
348	121104	42144192	18·6547581	7·033850
349	121801	42508549	18·6815417	7·040581
350	122500	42875000	18·7082869	7·047208
351	123201	43243551	18·7349940	7·054003
352	123904	43614208	18·7616630	7·060696
353	124609	43986977	18·7882942	7·067376
354	125316	44361864	18·8148877	7·074043
355	126025	44738875	18·8414437	7·080698
356	126736	45118016	18·8679623	7·087341
357	127449	45499293	18·8944436	7·093970
358	128164	45882712	18·9208879	7·100588
359	128881	46268279	18·9472953	7·107193
360	129600	46656000	18·9736660	7·113786

Number	Square.	Cube.	Square Root.	Cube Root.
361	130321	47045881	19·0000000	7·120367
362	131044	47437928	19·0262976	7·126935
363	131769	47832147	19·0525589	7·133492
364	132496	48228544	19·0787840	7·140037
365	133225	48627125	19·1049732	7·146569
366	133956	49027896	19·1311265	7·153090
367	134689	49430863	19·1572441	7·159599
368	135424	49836032	19·1833261	7·166095
369	136161	50243409	19·2093727	7·172580
370	136900	50653000	19·2353841	7·179054
371	137641	51064811	19·2613603	7·185516
372	138384	51478848	19·2873015	7·191966
373	139129	51895117	19·3132079	7·198405
374	139876	52313624	19·3390796	7·204832
375	140625	52734375	19·3649167	7·211247
376	141376	53157376	19·3907194	7·217652
377	142129	53582633	19·4164878	7·224045
378	142884	54010152	19·4422221	7·230427
379	143641	54439939	19·4679223	7·236797
380	144400	54872000	19·4935887	7·243156
381	145161	55306341	19·5192213	7·249504
382	145924	55742968	19·5448203	7·255841
383	146689	56181887	19·5703858	7·262167
384	147456	56623104	19·5959179	7·268482
385	148225	57066625	19·6214169	7·274786
386	148996	57512456	19·6468827	7·281079
387	149769	57960603	19·6723156	7·287362
388	150544	58411072	19·6977156	7·293633
389	151321	58863869	19·7230829	7·299893
390	152100	59319000	19·7484177	7·306143
391	152881	59776471	19·7737199	7·312383
392	153664	60236288	19·7989899	7·318611
393	154449	60698457	19·8242276	7·324829
394	155236	61162984	19·8494332	7·331037
395	156025	61629875	19·8746069	7·337234
396	156816	62099136	19·8997487	7·343420
397	157609	62570773	19·9248588	7·349596
398	158404	63044792	19·9499373	7·355762
399	159201	63521199	19·9749844	7·361917
400	160000	64000000	20·0000000	7·368063
401	160801	64481201	20·0249844	7·374198
402	161604	64964808	20·0499377	7·380322
403	162409	65450827	20·0748599	7·386437
404	163216	65939264	20·0997512	7·392542
405	164025	66430125	20·1246118	7·398636

Number	Square.	Cube.	Square Root.	Cube Root.
406	164836	66923416	20·1494417	7·404720
407	165649	67419143	20·1742410	7·410794
408	166464	67911312	20·1990099	7·416859
409	167281	68417929	20·2237484	7·422914
410	168100	68921000	20·2484567	7·428958
411	168921	69426531	20·2731349	7·434993
412	169744	69934528	20·2977831	7·441018
413	170569	70444997	20·3224014	7·447033
414	171396	70957944	20·3469899	7·453039
415	172225	71473375	20·3715488	7·459036
416	173056	71991296	20·3960781	7·465022
417	173889	72511713	20·4205779	7·470999
418	174724	73034632	20·4450483	7·476966
419	175561	73560059	20·4694895	7·482924
420	176400	74088000	20·4939015	7·488872
421	177241	74618461	20·5182845	7·494810
422	178084	75151448	20·5426386	7·500740
423	178929	75686967	20·5669638	7·506660
424	179776	76225024	20·5912603	7·512571
425	180625	76765625	20·6155281	7·518473
426	181476	77308776	20·6397674	7·524365
427	182329	77854483	20·6639783	7·530248
428	183184	78402752	20·6881609	7·536121
429	184041	78953589	20·7123152	7·541986
430	184900	79507000	20·7364414	7·547841
431	185761	80062991	20·7605395	7·553688
432	186624	80621568	20·7846097	7·559525
433	187489	81182737	20·8086520	7·565353
434	188356	81746504	20·8326667	7·571173
435	189225	82312875	20·8566536	7·576984
436	190096	82881856	20·8806130	7·582786
437	190969	83453453	20·9045450	7·588579
438	191844	84027672	20·9284495	7·594363
439	192721	84604519	20·9523268	7·600138
440	193600	85184000	20·9761770	7·605905
441	194481	85766121	21·0000000	7·611662
442	195364	86350888	21·0237960	7·617411
443	196249	86938307	21·0475652	7·623151
444	197136	87528384	21·0713075	7·628883
445	198025	88121125	21·0950231	7·634606
446	198916	88716536	21·1187121	7·640321
447	199809	89314623	21·1423745	7·646027
448	200704	89915392	21·1660105	7·651725
449	201601	90518849	21·1896201	7·657414
450	202500	91125000	21·2132034	7·663094

Number	Square.	Cube.	Square Root.	Cube Root.
451	203401	91733851	21·2367606	7·668766
452	204304	92345408	21·2602916	7·674430
453	205209	92959677	21·2837967	7·680085
454	206116	93576664	21·3072758	7·685732
455	207025	94196375	21·3307290	7·691371
456	207936	94818816	21·3541565	7·697002
457	208849	95443993	21·3775583	7·702624
458	209764	96071912	21·4009346	7·708238
459	210681	96702579	21·4242853	7·713844
460	211600	97336000	21·4476106	7·719442
461	212521	97972181	21·4709106	7·725032
462	213444	98611128	21·4941853	7·730614
463	214369	99252847	21·5174348	7·736187
464	215296	99897344	21·5406592	7·741753
465	216225	100544625	21·5638587	7·747310
466	217156	101194696	21·5870331	7·752860
467	218089	101847563	21·6101828	7·758402
468	219024	102503232	21·6333077	7·763936
469	219961	103161709	21·6564078	7·769462
470	220900	103823000	21·6794834	7·774980
471	221841	104487111	21·7025344	7·780490
472	222784	105154048	21·7255610	7·785992
473	223729	105823817	21·7485632	7·791487
474	224676	106496424	21·7715411	7·796974
475	225625	107171875	21·7944947	7·802453
476	226576	107850176	21·8174242	7·807925
477	227529	108531333	21·8403297	7·813389
478	228484	109215352	21·8632111	7·818845
479	229441	109902239	21·8860686	7·824294
480	230400	110592000	21·9089023	7·829735
481	231361	111284641	21·9317122	7·835168
482	232324	111980168	21·9544984	7·840594
483	233289	112678587	21·9772610	7·846013
484	234256	113379904	22·0000000	7·851424
485	235225	114084125	22·0227155	7·856828
486	236196	114791256	22·0454077	7·862224
487	237169	115501303	22·0680765	7·867613
488	238144	116214272	22·0907220	7·872994
489	239121	116930169	22·1133444	7·878368
490	240100	117649000	22·1359436	7·883734
491	241081	118370771	22·1585198	7·889094
492	242064	119095488	22·1810730	7·894446
493	243049	119823157	22·2036033	7·899791
494	244036	120553784	22·2261108	7·905129
495	245025	121287375	22·2485955	7.910460

Number	Square.	Cube.	Square Root.	Cube Root.
496	246016	122023936	22·2710575	7·915784
497	247009	122763473	22·2934968	7·921100
498	248004	123505992	22·3159136	7·926408
499	249001	124251499	22·3383079	7·931710
500	250000	125000000	22·3606798	7·937005
501	251001	125751501	22·3830293	7·942293
502	252004	126506008	22·4053565	7·947573
503	253009	127263527	22·4276615	7·952847
504	254016	128024064	22·4499443	7·958114
505	255025	128787625	22·4722051	7·963374
506	256036	129554216	22·4944438	7·968627
507	257049	130323843	22·5166605	7·973873
508	258064	131096512	22·5388553	7·979112
509	259081	131872229	22·5610283	7·984344
510	260100	132651000	22·5831796	7·989569
511	261121	133432831	22·6053091	7·994788
512	262144	134217728	22·6274170	8·000000
513	263169	135005697	22·6495033	8·005205
514	264196	135796744	22·6715681	8·010403
515	265225	136590875	22·6936114	8·015595
516	266256	137388096	22·7156334	8·020779
517	267289	138188413	22·7376340	8·025957
518	268324	138991832	22·7596134	8·031129
519	269361	139798359	22·7815715	8·036293
520	270400	140608000	22·8035085	8·041451
521	271441	141420761	22·8254244	8·046603
522	272484	142236648	22·8473193	8·051748
523	273529	143055667	22·8691933	8·056886
524	274576	143877824	22·8910463	8·062018
525	275625	144703125	22·9128785	8·067143
526	276676	145531576	22·9346899	8·072262
527	277729	146363183	22·9564806	8·077374
528	278784	147197952	22·9782506	8·082480
529	279841	148035889	23·0000000	8·087579
530	280900	148877000	23·0217289	8·092672
531	281961	149721291	23·0434372	8·097758
532	283024	150568768	23·0651252	8·102838
533	284089	151419437	23·0867928	8·107912
534	285156	152273304	23·1084400	8·112980
535	286225	153130375	23·1300670	8·118041
536	287296	153990656	23·1516738	8·123096
537	288369	154854153	23·1732605	8·128144
538	289444	155720872	23·1948270	8·133186
539	290521	156590819	23·2163735	8·138223
540	291600	157464000	23·2379001	8·143253

Number	Square.	Cube.	Square Root.	Cube Root.
541	292681	158340421	23·2594067	8·148276
542	293764	159220088	23·2808935	8·153293
543	294849	160103007	23·3023604	8·158304
544	295936	160989184	23·3238076	8·163309
545	297025	161878625	23·3452351	8·168308
546	298116	162771336	23·3666429	8·173302
547	299209	163667323	23·3880311	8·178289
548	300304	164566592	23·4093998	8·183269
549	301401	165469149	23·4307490	8·188244
550	302500	166375000	23·4520788	8·193212
551	303601	167284151	23·4733892	8·198175
552	304704	168196608	23·4946802	8·203131
553	305809	169112377	23·5159520	8·208082
554	306916	170031464	23·5372046	8·213027
555	308025	170953875	23·5584380	8·217965
556	309136	171879616	23·5796522	8·222898
557	310249	172808693	23·6008474	8·227825
558	311364	173741112	23·6220236	8·232746
559	312481	174676879	23·6431808	8·237661
560	313600	175616000	23·6643191	8·242570
561	314721	176558481	23·6854386	8·247474
562	315844	177504328	23·7065392	8·252371
563	316969	178453547	23·7276210	8·257263
564	318096	179406144	23·7486842	8·262149
565	319225	180362125	23·7697286	8·267029
566	320356	181321496	23·7907545	8·271903
567	321489	182284263	23·8117618	8·276772
568	322624	183250432	23·8327506	8·281635
569	323761	184220009	23·8537209	8,286493
570	324900	185193000	23·8746728	8·291344
571	326041	186169411	23·8956063	8·296190
572	327184	187149248	23·9165215	8·301030
573	328329	188132517	23·9374184	8·305865
574	329476	189119224	23·9582971	8·310694
575	330625	190109375	23·9791576	8·315517
576	331776	191102976	24·0000000	8·320335
577	332929	192100033	24·0208243	8·325147
578	334084	193100552	24·0416306	8·329954
579	335241	194104539	24·0624188	8·334755
580	336400	195112000	24·0831892	8·339551
581	337561	196122941	24·1039416	8·344341
582	338724	197137368	24·1246762	8·349125
583	339889	198155287	24·1453929	8·353904
584	341056	199176704	24·1660919	8·358678
585	342225	200201625	24·1867732	8·363446

Number	Square.	Cube.	Square Root.	Cube Root.
586	343396	201230056	24·2074369	8·368209
587	344569	202262003	24·2280829	8·372966
588	345744	203297472	24·2487113	8·377718
589	346921	204336469	24·2693222	8·382465
590	348100	205379000	24·2899156	8·387206
591	349281	206425071	24·3104916	8·391942
592	350464	207474688	24·3310501	8·396673
593	351649	208527857	24·3515913	8·401398
594	352836	209584584	24·3721152	8·406118
595	354025	210644875	24·3926218	8·410832
596	355216	211708736	24·4131112	8·415541
597	356409	212776173	24·4335834	8·420245
598	357604	213847192	24·4540385	8·424944
599	358801	214921799	24·4744765	8·429638
600	360000	216000000	24·4948974	8·434327
601	361201	217081801	24·5153013	8·439009
602	362404	218167208	24·5356883	8·443687
603	363609	219256227	24·5560583	8·448360
604	364816	220348864	24·5764115	8·453027
605	366025	221445125	24·5967478	8·457689
606	367236	222545016	24·6170673	8·462347
607	368449	223648543	24·6373700	8·466999
608	369664	224755712	24·6576560	8·471647
609	370881	225866529	24·6779254	8·476289
610	372100	226981000	24·6981781	8·480926
611	373321	228099131	24·7184142	8·485557
612	374544	229220928	24·7386338	8·490184
613	375769	230346397	24·7588368	8·494806
614	376996	231475544	24·7790234	8·499423
615	378225	232608375	24·7991935	8·504034
616	379456	233744896	24·8193473	8·508641
617	380689	234885113	24·8394847	8·513243
618	381924	236029032	24·8596058	8·517840
619	383161	237176659	24·8797106	8·522432
620	384400	238328000	24·8997992	8·527018
621	385641	239483061	24·9198716	8·531600
622	386884	240641848	24·9399278	8·536177
623	388129	241804367	24·9599679	8·540749
624	389376	242970624	24·9799920	8·545317
625	390625	244140625	25·0000000	8·549879
626	391876	245314376	25·0199920	8·554437
627	393129	246491883	25·0399681	8·558990
628	394384	247673152	25·0599282	8·563537
629	395641	248858189	25·0798724	8·568080
630	396900	250047000	25·0998008	8·572618

Number	Square.	Cube.	Square Root.	Cube Root.
631	398161	251239591	25·1197134	8·577152
632	399424	252435968	25·1396102	8·581680
633	400689	253636137	25·1594913	8·586204
634	401956	254840104	25·1793566	8·590723
635	403225	256047875	25·1992063	8·595238
636	404496	257259456	25·2190404	8·599747
637	405769	258474853	25·2388589	8·604252
638	407044	259694072	25·2586619	8·608752
639	408321	260917119	25·2784493	8·613248
640	409600	262144000	25·2982213	8·617738
641	410881	263374721	25·3179778	8·622224
642	412164	264609288	25·3377189	8·626706
643	413449	265847707	25·3574447	8·631183
644	414736	267089984	25·3771551	8·635655
645	416025	268336125	25·3968502	8·640122
646	417316	269586136	25·4165301	8·644585
647	418609	270840023	25·4361947	8·649043
648	419904	272097792	25·4558441	8·653497
649	421201	273359449	25·4754784	8·657946
650	422500	274625000	25·4950076	8·662301
651	423801	275894451	25·5147016	8·666831
652	425104	277167808	25·5342907	8·671266
653	426409	278445077	25·5538647	8·675697
654	427716	279726264	25·5734237	8·680123
655	429025	281011375	25·5929678	8·684545
656	430336	282300416	25·6124969	8·688963
657	431649	283593393	25·6320112	8·693376
658	432964	284890312	25·6515107	8·697784
659	434281	286191179	25·6709953	8·702188
660	435600	287496000	25·6904652	8·706587
661	436921	288804781	25·7099203	8·710982
662	438244	290117528	25·7203607	8·715373
663	439569	291434247	25·7487864	8·719759
664	440896	292754944	25·7681975	8·724141
665	442225	294079625	25·7875939	8·728518
666	443556	295408296	25·8069758	8·732891
667	444889	296740963	25·8263431	8·737260
668	446224	298077632	25·8456960	8·741624
669	447561	299418309	25·8650343	8·745984
670	448900	300763000	25·8843582	8·750340
671	450241	302111711	25·9036677	8·754691
672	451584	303464448	25·9229628	8·759038
673	452929	304821217	25·9422435	8·763380
674	454276	306182024	25·9615100	8·767719
675	455625	307546875	25·9807621	8·772053

Number	Square.	Cube.	Square Root.	Cube Root.
676	456976	308915776	26·0000000	8·776382
677	458329	310288733	26·0192237	8·780708
678	459684	311665752	26·0384331	8·785029
679	461041	313046839	26·0576284	8.789346
680	462400	314432000	26·0768096	8·793659
681	463761	315821241	26·0959767	8·797967
682	465124	317214568	26·1151297	8·802272
683	466489	318611987	26·1342687	8·806572
684	467856	320013504	26·1533937	8·810868
685	469225	321419125	26·1725047	8·815159
686	470596	322828856	26·1916017	8·819447
687	471969	324242703	26·2106848	8·823730
688	473344	325660672	26·2297541	8·828009
689	474721	327082769	26·2488095	8·832285
690	476100	328509000	26·2678511	8·836556
691	477481	329939371	26·2868789	8·840822
692	478864	331373888	26·3058929	8·845085
693	480249	332812557	26·3248932	8·849344
694	481636	334255384	26·3438797	8·853598
695	483025	335702375	26·3628527	8·857849
696	484416	337153536	26·3818119	8·862095
697	485809	338608873	26·4007576	8·866337
698	487204	340068392	26·4196896	8·870575
699	488601	341532099	26·4386081	8·874809
700	490000	343000000	26·4575131	8·879040
701	491401	344472101	26·4764046	8·883266
702	492804	345948008	26·4952826	8·887488
703	494209	347428927	26·5141472	8·891706
704	495616	348913664	26·5329983	8·895920
705	497025	350402625	26·5518361	8·900130
706	498436	351895816	26·5706605	8·904336
707	499849	353393243	26·5894716	8·908538
708	501264	354894912	26·6082694	8·912736
709	502681	356400829	26·6270539	8·916931
710	504100	357911000	26·6458252	8·921121
711	505521	359425431	26·6645833	8·925307
712	506944	360944128	26·6833281	8·929490
713	508369	362467097	26·7020598	8·933668
714	509796	363994344	26·7207784	8·937843
715	511225	365525875	26·7394839	8·942014
716	512656	367061696	26·7581763	8·946180
717	514089	368601813	26·7768557	8·950343
718	515524	370146232	26·7955220	8·954502
719	516961	371694959	26·8141754	8·958658
720	518400	373248000	26·8328157	8·962809

Number	Square.	Cube.	Square Root.	Cube Root.
721	519841	374805361	26·8514432	8·966957
722	521284	376367048	26·8700577	8·971100
723	522729	377933067	26·8886593	8·975240
724	524176	379503424	26·9072481	8·979376
725	525625	381078125	26·9258240	8·983508
726	527076	382657176	26·9443872	8·987637
727	528529	384240583	26·9629375	8·991762
728	529984	385828352	26·9814751	8·995883
729	531441	387420489	27·0000000	9·000000
730	532900	389017000	27·0185122	9·004113
731	534361	390617891	27·0370117	9·008222
732	535824	392223168	27·0554985	9·012328
733	537289	393832837	27·0739727	9·016430
734	538756	395446904	27·0924344	9·020529
735	540225	397065375	27·1108834	9·024623
736	541696	398688256	27·1293199	9·028714
737	543169	400315553	27·1477439	9·032802
738	544644	401947272	27·1661554	9·036885
739	546121	403583419	27·1845544	9·040965
740	547600	405224000	27·2029410	9·045041
741	549081	406869021	27·2213152	9·049114
742	550564	408518488	27·2396769	9·053183
743	552049	410172407	27·2580263	9·057248
744	553536	411830784	27·2763634	9·061309
745	555025	413493625	27·2946881	9·065367
746	556516	415160936	27·3130006	9·069422
747	558009	416832723	27·3313007	9.073472
748	559504	418508992	27·3495887	9·077519
749	561001	420189749	27·3678644	9·081563
750	562500	421875000	27·3861279	9·085603
751	564001	423564751	27·4043792	9·089639
752	565504	425259008	27·4226184	9·093672
753	567009	426957777	27·4408455	9·097701
754	568516	428661064	27·4590604	9·101726
755	570025	430368875	27·4772633	9·105748
756	571536	432081216	27·4954542	9·109766
757	573049	433798093	27·5136330	9·113781
758	574564	435519512	27·5317998	9·117793
759	576081	437245479	27·5499546	9·121801
760	577600	438976000	27·5680975	9·125805
761	579121	440711081	27·5862284	9·129806
762	580644	442450728	27·6043475	9·133803
763	582169	444194947	27·6224546	9·137797
764	583696	445943744	27·6405499	9·141788
765	585225	447697125	27·6586334	9·145774

Number	Square.	Cube.	Square Root	Cube Root.
766	586756	449455096	27·6767050	9·149757
767	588289	451217663	27·6947648	9·153737
768	589824	452984832	27·7128129	9·157713
769	591361	454756609	27·7308492	9·161686
770	592900	456533000	27·7488739	9·165656
771	594441	458314011	27·7668868	9·169622
772	595984	460099648	27·7848880	9·173585
773	597529	461889917	27·8028775	9·177544
774	599076	463684824	27·8208555	9·181500
775	600625	465484375	27·8388218	9·185452
776	602176	467288576	27·8567766	9·189401
777	603729	469097433	27·8747197	9·193347
778	605284	470910952	27·8926514	9·197289
779	606841	472729139	27·9105715	9·201228
780	608400	474552000	27·9284801	9·205164
781	609961	476379541	27·9463772	9·209096
782	611524	478211768	27·9642629	9·213025
783	613089	480048687	27·9821372	9·216950
784	614656	481890304	28·0000000	9·220872
785	616225	483736025	28·0178515	9·224791
786	617796	485587656	28·0356915	9·228706
787	619369	487443403	28·0535203	9·232618
788	620944	489303872	28·0713377	9·237527
789	622521	491169069	28·0891438	9·240433
790	624400	493039000	28·1069386	9·244335
791	625681	494913671	28·1247222	9·248234
792	627264	496793088	28·1424946	9·252130
793	628849	498677257	28·1602557	9.256022
794	630436	500566184	28·1780056	9·259911
795	632025	502459875	28·1957444	9·263797
796	633616	504358336	28·2134720	9·267679
797	635209	506261573	28·2311884	9·271559
798	636804	508169592	28·2488938	9·275435
799	638401	510082399	28·2665881	9·279308
800	640000	512000000	28·2842712	9·283177
801	641601	513922401	28·3019434	9·287044
802	643204	515849608	28·3196045	9·290907
803	644809	517781627	28·3372546	9·294767
804	646416	519718464	28·3548938	9·298623
805	648025	521660125	28·3725219	9·302477
806	649636	523606616	28·3901391	9·306327
807	651249	525557943	28·4077454	9·310175
808	652864	527514112	28·4253408	9·314019
809	654481	529475129	28·4429253	9·317859
810	656100	531441000	28·4604989	9·321697

Number	Square.	Cube.	Square Root.	Cube Root.
811	657721	533411731	28·4780617	9·325532
812	659344	535387328	28·4956137	9·329363
813	660969	537366797	28·5131549	9·333191
814	662596	539353144	28·5306852	9·337016
815	664225	541343375	28·5482048	9·340838
816	665856	543338496	28·5657137	9·344657
817	667489	545338513	28·5832119	9·348473
818	669124	547343432	28·6006993	9·352285
819	670761	549353259	28·6181760	9·356095
820	672400	551368000	28·6356421	9·359901
821	674041	553387661	28·6530976	9·363704
822	675684	555412248	28·6705424	9·367505
823	677329	557441767	28·6879766	9·371302
824	678976	559476224	28·7054002	9·375096
825	680625	561515625	28·7228132	9·378887
826	682276	563559976	28·7402157	9·382675
827	683929	565609283	28·7576077	9·386460
828	685584	567663552	28·7749891	9·390241
829	687241	569722789	28·7923601	9·394020
830	688900	571787000	28·8097206	9·397796
831	690561	573856191	28·8270706	9·401569
832	692224	575930368	28·8444102	9·405338
833	693889	578009537	28·8617394	9·409105
834	695556	580093704	28·8790582	9·412869
835	697225	582182875	28·8963666	9·416630
836	698896	584277056	28·9136646	9·420387
837	700569	586376253	28·9309523	9·424141
838	702244	588480472	28·9482297	9·427893
839	703921	590589719	28·9654967	9·431642
840	705600	592704000	28·9827535	9·435388
841	707281	594823321	29·0000000	9·439130
842	708964	596947688	29·0172363	9·442870
843	710649	599077107	29·0344623	9·446607
844	712336	601211584	29·0516781	9·450341
845	714025	603351125	29·0688837	9·454071
846	715716	605495736	29·0800791	9·457799
847	717409	607645423	29·1032644	9·461524
848	719104	609800192	29·1204396	9·465247
849	720801	611960049	29·1376046	9·468966
850	722500	614125000	29·1547595	9·472682
851	724201	616295051	29·1719043	9·476395
852	725904	618470208	29·1890390	9·480106
853	727609	620650477	29·2061637	9·483813
854	729316	622835864	29·2232784	9·487518
855	731025	625026375	29·2403830	9·491219

Number	Square.	Cube.	Square Root.	Cube Root.
856	732736	627222016	29·2574777	9·494918
857	734449	629422793	29·2745623	9·498614
858	736164	631628712	29·2916370	9·502307
859	737881	633839779	29·3087018	9·505998
860	739600	636056000	29·3257566	9·509685
861	741321	638277381	29·3428015	9·513369
862	743044	640503928	29·3598365	9·517051
863	744769	642735647	29·3768616	9·520730
864	746496	644972544	29·3938769	9·524406
865	748225	647214625	29·4108823	9·528079
866	749956	649461896	29·4278779	9·531749
867	751689	651714363	29·4448637	9·535417
868	753424	653972032	29·4618397	9·539081
869	755161	656234909	29·4788059	9·542743
870	756900	658503000	29·4957624	9·546402
871	758641	660776311	29·5127091	9·550058
872	760384	663054848	29·5296401	9·553712
873	762129	665338617	29·5465734	9·557363
874	763876	667627624	29·5634910	9·561010
875	765625	669921875	29·5803989	9·564655
876	767376	672221376	29·5972972	9·568297
877	769129	674526133	29·6141858	9·571937
878	770884	676836152	29·6310648	9·575574
879	772641	679151439	29·6479325	9·579208
880	774400	681472000	29·6647939	9·582839
881	776161	683797841	29·6816442	9·586468
882	777924	686128968	29·6984848	9·590093
883	779689	688465387	29·7153159	9·593716
884	781456	690807104	29·7321375	9·597337
885	783225	693154125	29·7489496	9·600954
886	784996	695506456	29·7657521	9·604569
887	786769	697864103	29·7825452	9·608181
888	788544	700227072	29·7993289	9·611791
889	790321	702595369	29·8161030	9·615397
890	792100	704969000	29·8328678	9·619001
891	793881	707347971	29·8496231	9·622603
892	795664	709732288	29·8663690	9·626201
893	797449	712121957	29·8831056	9·629797
894	799236	714516984	29·8998328	9·633390
895	801025	716917375	29·9165506	9·636981
896	802816	719323136	29·9332591	9·640569
897	804609	721734273	29·9499583	9·644154
898	806404	724150792	29·9666481	9·647736
899	808201	726572699	29·9833287	9·651316
900	810000	729000000	30·0000000	9·654893

Number	Square.	Cube.	Square Root.	Cube Root.
901	811801	731432701	30·0166620	9·658468
902	813604	733870808	30·0333148	9·662040
903	815409	736314327	30·0499584	9·665609
904	817216	738763264	30·0665928	9·669176
905	819025	741217625	30·0832179	9·672740
906	820836	743677416	30·0998339	9·676301
907	822649	746142643	30·1164407	9.679860
908	824464	748613312	30·1330383	9·683416
909	826281	751089429	30·1496269	9·686970
910	828100	753571000	30·1632063	9·690521
911	829921	756058031	30·1827765	9·694069
912	831744	758550528	30·1993377	9·697615
913	833569	761048497	30·2158899	9·701158
914	835396	763551944	30·2324329	9·704698
915	837225	766060875	30·2489669	9·708236
916	839056	768575296	30·2654919	9·711772
917	840889	771095213	30·2820079	9·715305
918	842724	773620632	30·2985148	9·718835
919	844561	776151559	30·3150128	9·722363
920	846400	778688000	30·3315018	9·725888
921	848241	781229961	30·3479818	9·729410
922	850084	783777448	30·3644529	9·732930
923	851929	786330467	30·3809151	9·736448
924	853776	788889024	30·3973683	9·739963
925	855625	791453125	30·4138127	9·743475
926	857476	794022776	30·4302481	9·746985
927	859329	796597983	30·4466747	9·750493
928	861184	799178752	30·4630924	9·753998
929	863041	801765089	30·4795013	9·757500
930	864900	804357000	30·4959014	9·761000
931	866761	806954491	30·5122926	9·764497
932	868624	809557568	30·5286750	9·767992
933	870489	812166237	30·5450487	9·771484
934	872356	814780504	30·5614136	9·774974
935	874225	817400375	30·5777697	9·778461
936	876096	820025856	30·5941171	9·782946
937	877969	822656953	30·6104557	9·785428
938	879844	825293672	30·6267857	9·788908
939	881721	827936019	30·6431069	9·792386
940	883600	830584000	30·6594194	9·795861
941	885481	833237621	30·6757233	9·799333
942	887364	835896888	30·6920185	9·802803
943	889249	838561807	30·7083051	9·806271
944	891136	841232384	30·7245830	9·809736
945	893025	843908625	30·7408523	9·813198

Number	Square.	Cube.	Square Root.	Cube Root.
946	894916	846590536	30·7571130	9·816659
947	896809	849278123	30·7733651	9·820117
948	898704	851971392	30·7896086	9·823572
949	900601	854670349	30·8058436	9·827025
950	902500	857375000	30·8220700	9·830475
951	904401	860085351	30·8382879	9·833923
952	906304	862801408	30·8544972	9·837369
953	908209	865523177	30·8706981	9·840812
954	910116	868250664	30·8868904	9·844253
955	912025	870983875	30·9030743	9·847692
956	913936	873722816	30·9192497	9·851128
957	915849	876467493	30·9354166	9·854561
958	917764	879217912	30·9515751	9·857992
959	919681	881974079	30·9677251	9·864421
960	921600	884736000	30·9838668	9·861848
961	923521	887503681	31·0000000	9·868272
962	925444	890277128	31·0161248	9·871694
963	927369	893056347	31·0322413	9·875113
964	929296	895841344	31·0483494	9·878530
965	931225	893682125	31·0644491	9·881945
966	933156	901428696	31·0805405	9·885357
967	935089	904231063	31·0966236	9·888767
968	937024	907039232	31·1126984	9·892174
969	938961	909853209	31·1287648	9·895580
970	940900	912673000	31·1448230	9·898983
971	942841	915498611	31·1608729	9·902383
972	944784	918330048	31·1769145	9·905781
973	946729	921167317	31·1929479	9·909177
974	948676	924010424	31·2089731	9·912571
975	950625	926859375	31·2249900	9·915962
976	952576	929714176	31·2409987	9·919351
977	954529	932574833	31·2569992	9·922738
978	956484	935441352	31·2729915	9·926122
979	958441	938313739	31·2889757	9·929504
980	960400	941192001	31·3049517	9·932883
981	962361	944076141	31.3209195	9·936261
982	964324	946966168	31·3368792	9·939636
983	966289	949862087	31·3528308	9·943009
984	968256	952763904	31·3687743	9·946379
985	970225	955671625	31·3847097	9·949747
986	972196	958585256	31·4006369	9·953113
987	974169	961504803	31·4165561	9·956477
988	976144	964430272	31·4324673	9·959839
989	978121	967361669	31·4483704	9·963198
990	980100	970299000	31·4642654	9·966554

Number	Square.	Cube.	Square Root.	Cube Root.
991	982081	973242271	31·4801525	9·969909
992	984064	976191488	31·4960315	9·973262
993	986049	979146657	31·5119025	9·976612
994	988036	982107784	31·5277655	9·979959
995	990025	985074875	31·5436206	9·983304
996	992016	988047936	31·5594677	9·986648
997	994009	991026973	31·5753068	9·989990
998	996004	994011992	31·5911380	9·993328
999	998001	997002999	31·6069613	9·996665

(54) With regard to the making of patterns of cast iron shafts, I beg leave to refer the reader to what I have said in Essay I, page 153 to 161, relative to the making of patterns for cast iron wheels, which is, in a great measure, applicable to those of shafts. Nor do I recollect any thing on this subject to add here—only I would remind the Millwright to make the allowance for contraction of metal of $\frac{1}{8}$ of an inch to the foot in the pattern.

(55) The following table contains the dimensions of shafts subject to torsion, and to considerable lateral pressure, as they were executed by a respectable Millwright. It will serve to shew the sizes of the parts as found in practice sufficiently strong, and may be found useful to compare with those which would be produced, calculating on the principles laid down in this Essay.

Column 6th, therefore, shews the diameters which these journals ought to have, were 400 used as the multiplier.—(See No. 21.)

TABLE OF SHAFTS.

Name.	1 Horses' Power.	2 Revolutions per minute.	3 Diameter of journals in inches.	4 Length of Shaft.	5 Section in the middle.	6 Diameters of journals by calculating from a Multiplier of 400.	Remarks.
Cast Iron Lying Shaft.	20	20	6 Inc.	11	7¾	7,368	Feathered Shafts.
	18	22	5¾	11	7	6,889	
	16	22	5¼	10.6	7	6,621	
	14	24	5	10	6¾	6,153	
	12	25	5	9.6	6	5,768	
	10	25	4¾	9	6¼	5,428	
	8	27	4	9	6	4,904	
	6	28	4¼	8.6	5¾	4,414	
	5	30	3⅞	8.6	5¼	4,061	
	4	32	3¾	8.	4¾	3,484	
	3	34	3¼	8.	4	1,203	Square Shafts.
	2	46	2¾	8.	3	2,802	
	1	40	2	8.	3¼	2,154	
Malleable Iron Lying Shaft.	6	28	3	8.6			
	5	30	3¼	8.			
	4	32	2	8.			
	3	34	2	8.			
	2	36	1¾	8.			
	1	40	1	7.6			

APPENDIX.

COHESIVE STRENGTH OF DIFFERENT METALS.

1. " We shall take for the measure of cohesion the number of pounds avoirdupois which are just sufficient to tear asunder a rod or bundle of one inch square. From this it will be easy to compute the strength corresponding to any other dimension.

" 1st, Metals.

		lbs.
" Gold ca[illegible]		20,000 24,000
Silver cast		40,000 43,000
Copper cast	Japan	19,500
	Barbary	22,000
	Hungary	31,000
	Anglesea	34,000
	Sweden	37,000

		lbs.
Iron cast		42,000 59,000
Iron bar	Ordinary	68,000
	Stirian	75,000
	Best Swedish and Russian	84,000
	Horse nails	71,000
Steel bar	Soft	120,000
	Razor Temper	150,000
Tin cast	Malacca	3,100
	Banca	3,600
	Block	3,800
	English block	5,200
	——— Grain	6,500
Lead cast		860
Regulus of Antimony		1,000
Zinc		2,600
Bismuth		2,900

"The only author who has put it in our power to judge of the propriety of his experiments is Muschenbroek. He has described his method of trial minutely, and it seems unexceptionable. The woods were all formed into slips fit for his apparatus, and part of the slip was cut away to a parallelopiped of $\frac{1}{5}$ of an inch square, and therefore $\frac{1}{25}$ of a square inch in section. The absolute strengths of a square inch were as follow:

	lbs.		lbs.
Locus tree	20,100	Pomegranate	9,750
Jujeb	18,500	Lemon	9,250
Beech Oak	17,300	Tamarind	8,750
Orange	15,500	Fir	8,330
Alder	13,900	Walnut	8,130
Elm	13,200	Pitch Pine	7,640
Mulberry	12,500	Quince	6,750
Willow	12,500	Cypress	6,000
Ash	12,000	Poplar	5,500
Plum	11,800	Cedar	4,880
Elder	18,000		

"Mr Muschenbroek has given a very minute detail of the experiments on the ash and the walnut, stating the weights which were required to tear asunder slips taken from the four sides of the tree, and on each side in a regular progression from the centre to the circumference. The numbers of this table corresponding to these two timbers may therefore be considered as the average of more than 50 trials made of each, and he says that all the others were made with the same care. We cannot therefore see any reason for not confiding in the results; yet they are considerably higher than those given by some other writers.

Mr Pitot says, on the authority of his own experiments, and of those of Mr Parent, that 60 pounds will just tear asunder a square line of sound oak, and that it will bear 50 with safety. This gives 8640 for the utmost strength of a square inch, which is much inferior to Muschenbroek's valuation.

" We may add to these

Ivory	16,280
Bone	5,250
Horn	8,860
Whalebone	8,500
Tooth of sea calf . . .	4,075

" The reader will surely observe that these numbers express something more than the utmost cohesion; for the weights are such as will very quickly, that is, in a minute or two, tear the rods asunder. It may be said in general that two thirds of these weights will sensibly impair the strength after a considerable while; and that one half is the utmost that can remain suspended at them without risk for ever; and it is this last allotment that the engineer should reckon upon in his constructions. There is however a considerable difference in this respect. Woods of a very straight fibre, such as fir, will be less impaired by any load which is not sufficient to break them immediately."

" (2) According to Mr Emerson, the load which may be safely suspended to an inch square is as follows:

	lbs.
Iron	76,400
Brass	35,600
Hempen rope	19,600
Ivory	15,700
Oak, box, yew, plum-tree	7,850
Elm, ash, beech	6,070
Walnut, plum	5,360
Red fir, holly, elder, plane, crab	5,000
Cherry, hazle	4,760
Alder, asp, birch, willows	4,290
Lead	430
Freestone	914

" He gives us a practical rule, that a cylinder whose diameter is *d* inches, loaded to one fourth of its absolute strength, will carry as follows:

Iron	135	Cwt.
Good rope	22	
Oak	14	
Fir	9	

" The rank which the different woods hold in this list of Mr Emerson's is very different from what we find in Muschenbroek's. But precise measures must not be expected in this matter. It is wonderful that in a matter of such unquestionable importance the public has not enabled some persons of judgment to make proper trials. They are beyond the abilities of private persons*.

" (3) Mr Banks (powers of machines, &c. p. 94) takes iron at an average to be four times as strong as oak, and 5¼ times as strong as deal or fir.

" (4) According to the experiments of various authors, the cohesive strength of a square inch of razor steel is about 150 thousand pounds, of soft steel, 120, of wrought iron 80, of cast iron 50, of good rope 20, of oak, beech, and willow wood, in the direction of their fibres 12, in fir 8, and of lead about three thousand pounds; the cohesive strength of a square inch of bricks 300, and of freestone 200; teak wood, the tectona grandis, is said to be still stronger than oak.

" The strength of different materials in resisting compression, is liable to great variation.

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* En. Brit. Art. Strength of Materials.

In steel and in willow wood, the cohesive and repulsive strength appear to be nearly equal. Oak will suspend much more than fir, but fir will support twice as much as oak, probably on account of the curvature of the fibres of oak. Freestone has been found to support about 2000 pounds for each square inch; oak, in some practical cases, more than 4000.

"The strongest wood of each tree is neither at the centre nor at the circumference, but in the middle between both; and in Europe it is generally thicker and firmer on the south-east side of the tree. Although iron is much stronger than wood, yet it is more liable to accidental imperfections; and when it fails it gives no warning of its approaching fracture. The equable equality of steel may be ascertained by corrosion in an acid, but there is no easy mode of detecting internal flaws in a bar of iron, and we can only rely on the honesty of the workmen for its soundness. Wood, when it is coupled, complains, or emits a sound, and after this, although it is much weakened, it may still retain strength enough to be of service*."

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* Young's Nat. Phil. Vol. I. p. 151.

THE following Letter, with which Mr JOHN ROBERTON, engineer, favoured me, as it relates to the subject of this Essay, and contains much useful matter, will, I hope, be acceptable to the reader :—

DEAR SIR,—In answer to your's, of the 7th, I shall throw out a few hints, which occur to me respecting the stress on gudgeons, shafts, &c. It is demonstrable, that by a judicious arrangement of wheels and pinions, in many cases much of the stress and friction may be avoided. This is a doctrine in practical mechanics of very great importance, when we consider, that in many cases almost the whole of the impelling power is expended in overcoming the friction of the machinery.

(1)—Let A, Fig. 1st, be an overshot water-wheel. Let the line *a b*, represent the line of direction of the centre of gravity of the water in the buckets. On the extremity of this wheel let there be a toothed wheel acting into the pinion B. It is obvious, that independently of the weight of the wheel, that the whole weight of the water will be supported by the axis C, and the teeth of the wheel at *d;* and the weight which each will sustain, will be in the ratio of C *e* to *e d.* That is, the weight on the gudgeon will be as the distance e d, while that on the teeth will be as *c e*; or if the radius of the toothed wheel were *c e*, the teeth would sustain the whole weight of the water, leaving no weight on the gudgeon but that of the wheel. Again, let the wheel B, be removed to C, it is evident that the gudgeon *c*, would have to sustain the weight of the water, and a great deal more. That is, the weight on the gudgeon would be increased as *e* to *e c*; so that it is evident, the stress and friction of the gudgeon must depend, in a great measure, on the size of the toothed wheel attached to the water wheel, and to the situation of the pinion B.

(2)—Again, let there be a wheel at A, Fig. 2d, fixed on the end or middle of a shaft working into the wheel or pinion B, of any size.—Let the teeth move in the direction *a b.* It is evident, that the gudgeon or shaft will tend to move in the contrary direction, that is, in the direction *c d*, and with the very same force that the teeth acts upon each other, as action and re-action are equal and in contrary directions, and for the same reason, the gudgeon of the wheel B, will tend to move in the direction *e f*, with the very same force, that is, the same force as the action of the teeth on each other. Indeed, in any single pair of wheels, of whatever form or construction, the tendency to break or bend the shaft, or cause friction, is the same as the action of the teeth on each other.

Now, if the above wheels were made of a double size, it is evident that the acting power on them would be only one half, and consequently, one half of the strain to break the shaft or cause friction.

(3)—In the case of an intervening wheel, the force or tendency to break the shaft depends on the situation of such intervening wheel. Thus, if it be placed in a direct line betwixt the centres of the conducted and conducting wheels, as at A, Fig. 3d, the shaft or gudgeon will tend to move in the line *a b* or *b a*, (according to the direction of the conductor) with double the force of the action of the teeth.

If the axis of the intervening wheel form a right angle with the axis of the other two wheels, the force to break the shaft will be to that of a pair of single wheels, or the action of the teeth on them, as the diagonal of a square is to one of its sides. That is, the direction of the force, and its intensity, will be represented by the diagonal, it being evident, from the well-known laws of mechanics, that by the action of the wheel B, Fig. 4, No. 1, the centre of the intervening wheel A, would tend to move in the line *a c*, and by its action in the wheel *e*, it would tend to move in the line *a d*. Let *a d* and *a c* represent the forces in these directions, complete the square or parallelogram, the diagonal of which will both represent the force and its direction. See also Fig. 4, No. 2, and No. 3.

(4)—On the other hand, if a wheel be placed betwixt two others, as A, Fig. 3, where A is the conductor, the teeth of which acts with equal force on each of the wheels B and C, it is evident, that the strain is wholly taken off the shaft, the forces being equal and opposite each other; and in Fig. 5, where A is supposed the conductor, and B and C the conducted wheels on which the teeth on each bear equally,—it is plain from what has been already stated, that the direction of the forces will be *d a* and *a c*, and letting *a d* and *a c* represent the direction and intensity of these forces, and compleating the parallelogram, we have the diagonal *a b* to represent the compound direction and intensity of the force.

(*5*)—There is another point that is worthy of attention. That is, the place on the shaft where the wheels are fixed. If a wheel is put on at the end of a shaft to drive any other or others, it is clear, that the whole or nearly the whole of the stress will be at the end of the shaft or journal. If the wheel be placed in the middle of the shaft, the strain to break it will be greatest at that part, but the force will be resisted equally by each journal. Indeed, on whatever part of the shaft a wheel is placed, at that very place is the greatest strain on the shaft, and the force on each journal will be in the inverse ratio of the distance of the wheel from the ends of the shaft.

In Fig. 6, let A represent a shaft, either upright or lying. If a single wheel, as B, fixed upon it, drive two pinions, as C D, directly opposite to each other, the shaft or journal will not be affected thereby—but if two wheels are placed on the shaft, driving each a pinion as in Fig. 7, both journals will be affected, and the greatest strain to break the shaft will be at the arms of the wheel, or close to them, and betwixt the arms and journal. In this case, the middle point of the shaft, as at A, being in a state of contrary pressure, and therefore no strain on that part, it may be considered as a lever on each side of A; this being the fixed point, and the greatest strain on the shaft being at the wheels. The strain on the journal will be inversely, as the distance of the wheels from the middle point A, it being understood that the wheels, pinions and resistances are all the same.

(6)—Again, if two wheels are placed on a shaft, and two pinions, both on the same side of the shaft, the journals must support the strain or force of the action of the teeth of both wheels, or indeed whatever number of wheels is on a shaft; and working into pinions on one side, the journal of that shaft must support a pressure equal to the whole of their action into the teeth of the wheels, or pinions into which they are connected; and the strain on the shaft to break it, depends on the situation of the wheels, as they may be placed on the shaft: It being understood, as formerly, that directly at the place where the wheels or pinion is fixed on the shaft, that it must support a pressure equal to the pressure of the teeth of the wheel in its corresponding wheel or pinion. And according as these pressures combine, or act in a contrary direction to each other, so must the strain on the journals, or tendency to break or bend the shaft, be.

Hence the importance of placing wheels and pinions, so that their actions on each other may be in contrary directions, or so as to avoid, as much as possible, the strain on the shaft or journals.

(7)—In lying shafts, where circumstances may require, it may be advisable to have the main or heaviest shaft on the lift of the wheels, as this will take off a considerable deal of their weight or friction on the journals. The other shaft will naturally be on the fall of the wheel (which is supposed in this case much lighter than on the main shaft), and will be prevented from jolting upwards, both its own weight, and a pressure equal to that on the teeth of the wheels, being supported by the journals: It being evident that whatever the wheels are, that the same pressure that is on the teeth of the wheels will be equally the same on both shafts, the one tending to increase the weight of the shaft on the journals, and the other to diminish it.

(8)—In the course of my business, I have made several observations on the foregoing subjects. Twenty years ago, a case of this nature occurred, of considerable importance, at the flour mill erected near the Slitt Mills of Partick. The water wheel, 16 feet diameter, makes about 10 or 11 revolutions per minute, driving, in general, two pair of stones; the pit wheel about 6½ feet diameter. The consequence was, that the machinery, framing, &c. were not of sufficient strength to bear the force applied by the pit wheels (though they were very strong, and well executed for ordinary cases), the shaft, framing, &c. was in a high state of tremour, the machinery working in a rough and straining manner, and the wooden teeth (5 inches broad, pitch about 4½), could not stand for any length of time. The mill was altered, by enlarging the pinion, to let the wheel run at 14 or 15 turns per minute; and I believe the pit wheel enlarged about one foot diameter; afterwards the mill wrought very well.

In common corn-mills, and many others, there is a great deal of the impelling power lost by the smallness of the pinions, &c. which causes a great friction on the shaft or spindle, as well as by the friction of the teeth. Were both wheel and pinion encreased in diameter, much advantage would arise, not only in saving of power, but in the tear and wear of machinery. It is on this account that a corn mill, constructed in the double way, other circumstances being the same, performs much more work than in the single way.

(10)—In short, more things of this nature take place in machinery, than the most of operative mechanics, or even philosophers, are aware of; and I firmly believe, that, with a knowledge of the affecting causes, a vast deal might be done in saving power. For example, in a cotton mill, the main shaft generally makes from 40 to 50 revolutions per minute. Let the weight of the shafts and machinery, the size of the journals, and the effect of the wheels on these shafts, and friction of journals, be taken into the account, on the one case, and let the main shaft be supposed to be reduced to half of the former velocity, that is, from 20 to 25 turns per minute, and let the strength of the shaft, together with a due proportion of wheels, be augmented, so as to preserve the same firmness in every part of the machinery as in the former, and that the ultimate part of the machinery may be brought to the same speed as formerly, it will be found on investigation, that in the last case there will be a considerable saving in the first, or impelling power.

It must not be lost sight of, that, by reducing the velocity of the main shaft, that a judicious increase of the diameters of the wheels thereon is absolutely necessary: Indeed, without strict attention to matters of that kind, the effect of any alteration that may be proposed or made is very precarious. A due regard to the proper diameter of wheels, according to the work they have to perform, is a matter of very great importance; and it will be found, on general enquiry, that in most cases of machinery, it would be prudent to have the wheels and pinions of large diameter; and, as I said before, by encreasing their size, the force, strain, and friction on the shafts and journals are diminished in the same ratio.

Suppose, in a mill, that a range of lying shafts, of 80 or 100 feet long together, with wheels, &c. fixed on them, weighed 7000lbs; the journals 4 inches diameter, making 46 turns per minute, the surface of the journal would at that rate move at about 50 feet per minute; and supposing that the friction was equal to one-third of the weight, we would have $7000 \div 3 = 2333 + 50 = 116650 \div 42000 = 2.77$; that is 2¾ horses power expended in overcoming the friction of these shafts*.

---

* That is, valuing the horse's power at 42000lbs. 1 foot per minute. See Essay on Teeth of Wheels, page 130—134.

Again, were these shafts and journals extended to 5 inches diameter, and their number of turns reduced to one half of the former; that is, to 23 turns per minute, the surface of the journal would move at the rate of 31 feet per minute, and the weight of the wheel and shafts encreased one half, or say, to 10,000lbs. we would have $10{,}000 \div 3 = 33.33 + 31 = 103323 \div 42.000 = 2.46$, that is, nearly 2½ horses power; so that in this last case there would be a saving of something more than three-tenths of a horse's power, and the machinery would be, in every respect, improved.

(11)—In a horse-gin, where a pinion is driven by a toothed wheel on the gin, the friction, or strain on the journals, depends on the situation of the horse-beam, at any moment of time when the lever or horse-beam is above or below the pinion, the friction on the shaft will be the least, and when in the opposite direction, the friction and force on the shaft will be the greatest, and the general friction will be as the size of the main wheel; that is, the smaller the main wheel is, the friction and force on the shaft will be the greater, and the larger this wheel is, the friction, &c. will be the less; it being understood that I mean the friction or strain on the gudgeon, or axle, on account of the re-action of the teeth of the wheel, and not the strain on the shaft, to twist it, nor any other friction or strain whatever.

(12)—In a water-wheel turning machinery, in many cases, it is most advisable to have the toothed wheel of the same diameter. In this case, whatever power or force is applied to the wheel, the same must be resisted by the teeth of the wheel; and it also follows, of course, that whatever is the size of the wheel, or pinion which is driven by the main wheel, that the very same strain is on the shaft; that is, the same as on the teeth; and the shaft must be sufficiently strong to withstand the pressure. But another circumstance occurs, that by increasing the size of the pinion, there must necessarily be increase of the diameter of the shaft to withstand the twist. The shaft by no means, however, keeps pace with the augmentation of the wheels, but is only as the cube-root; for instance, a water-wheel of 16 feet diameter would work very ill into a pinion of 12 inches, suppose its shaft or journal to be 3 inches diameter: Again, let the pinion be increased to 2 feet diameter, a shaft of 3¾ inches will be as sufficient to withstand the twist, the friction on the journals will be much less, and the strength to resist the strain will also be much increased. I do not see how general rules can be laid down with accuracy in these things. It is the particular circumstance of the case that will guide a mechanic in the construction of any piece of machinery; and if he be not well acquainted with these, it cannot be expected that his schemes will be well arranged.

The above observations are no doubt at variance with the opinions of those who are continually holding up the simplification of machinery: Instance Mr Fenwick's "Essay on the Simplification of Machinery." In many cases it is prudent to make machinery of a more complex nature than it is sometimes constructed; and in order that it may be easier driven, that the tear and wear may be lessened, as well as the ultimate expence and trouble of attending it.

The foregoing are only a few general hints. To go minutely into the subject would require more time and thought than I could at present bestow; but if any thing is in them from which you may draw any useful inference, it will be very agreeable to me. I am, Sir, your's respectfully, JOHN ROBERTON.

Tradestown, Glasgow,
12th Sept. 1809.

say to which Mr Roberton in the
alludes, Mr Fenwick infers,
most perfect machine is that
ewest moving parts.” But
ave been misled here
ze. Simplicity is, no
ble quality in a machine,
obtained without making a
r or of durability. A sledge
ng parts, and in that sense is
than a wheel-carriage, yet no one
could say that a sledge is more per-

or, perhaps, does the *simplicity* of a ma-ine consist strictly in having *few moving parts.* If the parts of a machine be few, they are perhaps more easily taken in by the eye at one view, which may make them more easily comprehended by the mind, and in that sense be more simple. But in machinery, the kind of simplicity at which we ought to aim, has more regard to the manner *of action* than to the number of the moving parts. Thus, for example, when a weight is to be raised, if one wheel worked by a screw be employed, the machine *consists of fewer parts* than two wheels and two pinions, applied to the same purpose. But in this last case, the *manner of action* is really more simple; for the action and resistance are directly opposed in the same line; whereas, in the case of the screw, the action is oblique, and experience shews that it has much more friction, and much less durability.

What is here said of *manner of action* is applicable in comparing machines, consisting each of trains of wheels and pinions; for the most durable, by longest maintaining the true figure of the teeth, will ultimately be the most simple in the manner of its action. It is evident, that when the teeth become much worn, that the *manner of action* becomes proportionably more oblique and less simple.

Respecting the durability of the wheel-work of mills, I beg leave to refer the reader to my Essay on the Teeth of Wheels, p. 121.

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TO

ESSAY II.

ON THE

SHAFTS OF MILLS.

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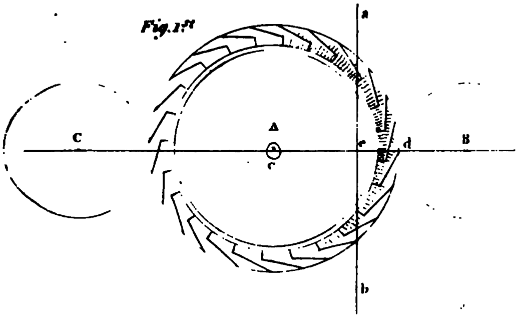
THE END.

APPENDIX ESSAY IIᵈ

Fig. 1st



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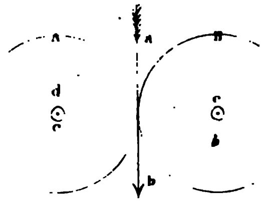

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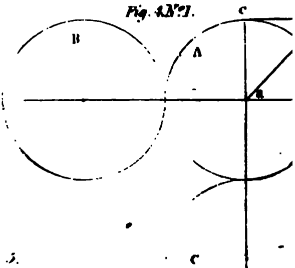

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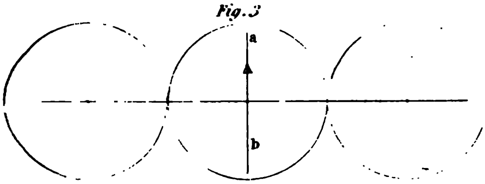

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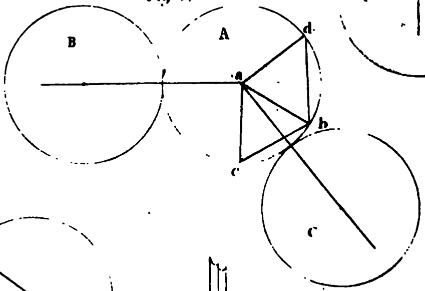

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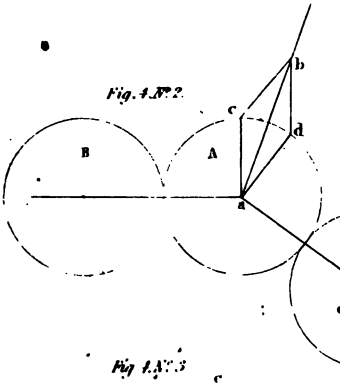

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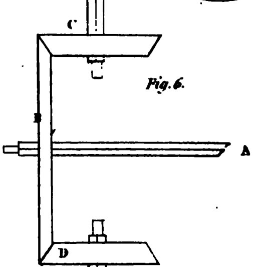

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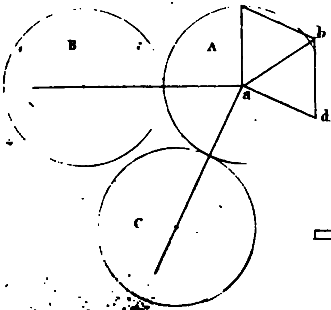

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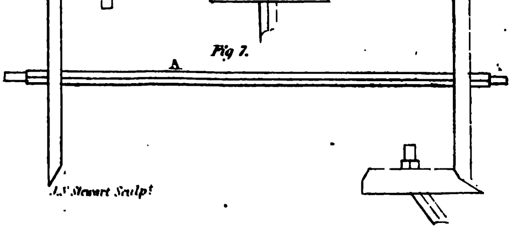

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PLATE XVII.

Fig. 1st

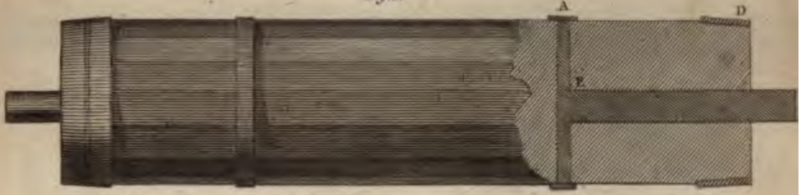

Fig. 1. N° 2.
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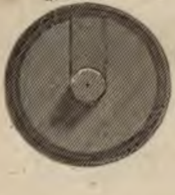

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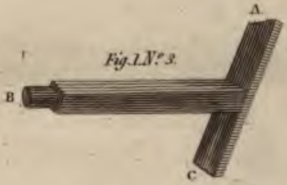

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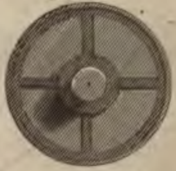
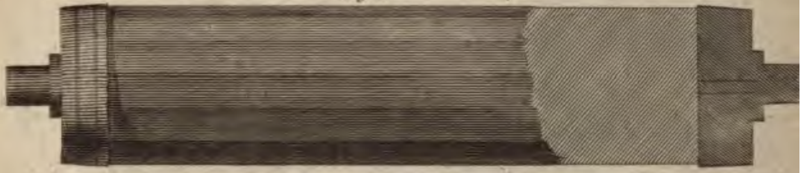


Fig. 2d



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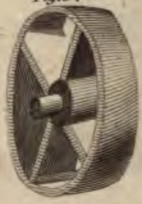

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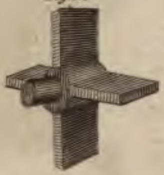

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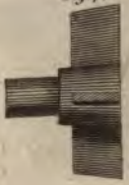

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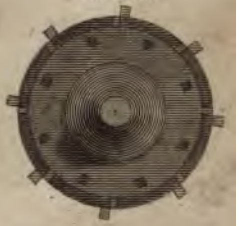

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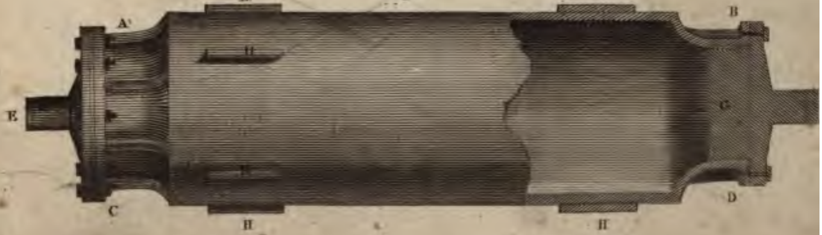

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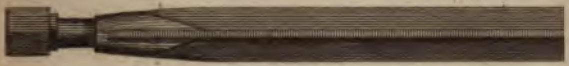
PLATE XVIII.

Fig. 7.



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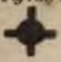

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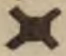

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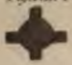

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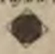

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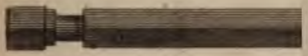

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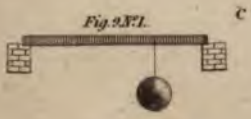

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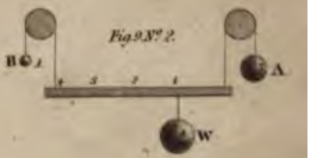

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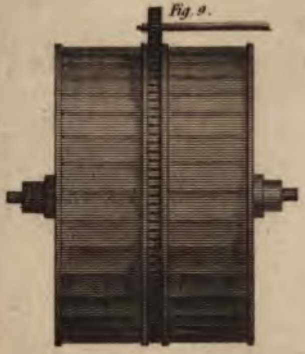

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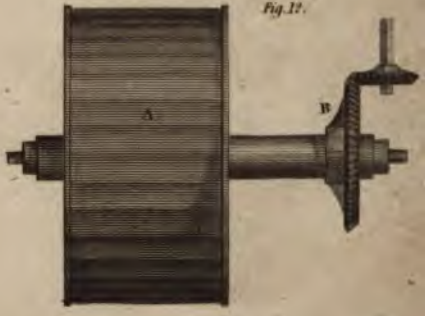

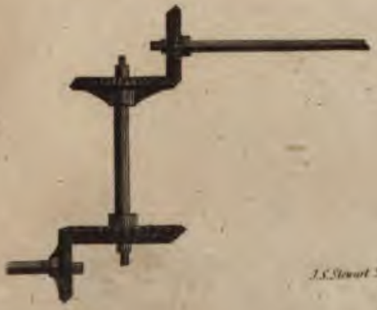

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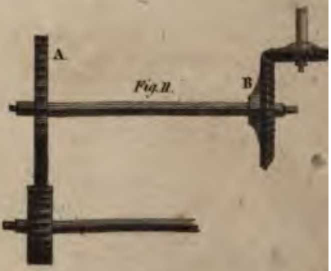

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